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Matrix Representation of Topological Changes in Metamorphic Mechanisms

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Metamorphic mechanisms form a class of mechanisms that has the facilities to change configuration from one kind to another with a resultant change in the number of effective links and mobility of movement. This paper develops formal matrix operations to describe the distinct topology of configurations found in a metamorphic mechanism and to complete transformation between them. A new way is hence introduced for modeling topological changes of metamorphic mechanisms in general. It introduces a new elimination E-elementary matrix together with a U-elementary matrix to form an EU-elementary matrix operation to produce the configuration transformation. The use of these matrix operations is demonstrated in both spherical and spatial metamorphic mechanisms, the mechanistic models taken from the industrial packaging operations of carton folding manipulation that stimulated this study. [DOI: 10.1115/1.1866159]

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1 A Metamorphic Mechanism and Its Topological Change

The essence of this paper was originally centered on a formalism for machine manipulation operations in folding a decorative carton in the context of product packaging [1,2]. A stiff, paneled carton with its creased hinges can be described [3–6] as a linkage mechanism [7–11], which in essence is just as relevant to any mechanism of a metamorphic kind, those with kinematic structure that changes.

An example of a mechanism that can be metamorphic [6] is a five-bar spherical linkage: one with five links joined in a loop by revolute joints whose axes are concurrent. The mechanism has two degrees of freedom and can change its topological configurations to one degree of freedom. The mechanism is also highly collapsible; it changes to a flattened configuration and has the property of adaptability [12,13]. A fully operable configuration is shown in Fig. 1.

A new configuration occurs when joint $\$_{4,5}$ lies in the plane containing $\$_{1,5}$ and $\$_{3,4}$, where link 4 overlaps link 5, which results in a topological change. At that position they may become self-attached or fixed by a device to become one link. The mechanism then becomes a four-bar spherical linkage as in Fig. 2, with joints $\$_{1,2}$, $\$_{2,3}$, $\$_{4,5}$ and $\$_{1,5}$, and the number of degrees of freedom is reduced to one.

2 Matrix Operations in Representing the Topological Changes

The topological structure [14–16] and link connectivity of a mechanism can be represented in a matrix form. In the matrix each link of the mechanism in its current configuration is identified by a number from 1 to n. The rows and columns of the matrix take these numbers in sequence. An entry of a connectivity between the ith link and jth link is given as element (i,j) in an adjacency matrix. When two links are connected, the entry of the corresponding row and column is given as 1. When two links are disconnected, the entry of the corresponding row and column is given as 0. This adjacency matrix defines a topological configuration of a mechanism during motion. The adjacency matrix of the five-bar configuration of the metamorphic mechanism in Fig. 1 is

$$\mathbf{A_0}=\begin{bmatrix}0&1&0&0&1\\1&0&1&0&0\\0&1&0&1&0\\0&0&1&0&1\\1&0&0&1&0\end{bmatrix}. \tag{1}$$

The connectivity changes into a four-bar configuration as in Fig. 2 when links 4 and 5 of the five-bar configuration are attached together. For this changed topological configuration the adjacency matrix becomes

$$\mathbf{A_1}=\begin{bmatrix}0&1&0&1\\1&0&1&0\\0&1&0&1\\1&0&1&0\end{bmatrix}. \tag{2}$$

As expected, this results in the change of both matrix elements and matrix order.

A topological change of the mechanism represented in these two matrices thus takes place. This topological change can be

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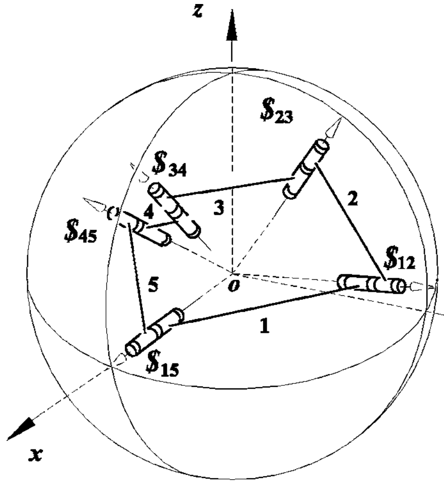

Fig. 1 Five-bar spherical metamorphic mechanism

derived from an analytical way using an EU-elementary matrix operation on the initial adjacency matrix, $\mathbf{A_0}$, to generate a new adjacency matrix, $\mathbf{A_1}$.

The EU-elementary matrix operation consists of the steps of applying a $\mathbf{U_{i,j}}$-elementary matrix [17] operation and an elimination $\mathbf{E_j}$-elementary matrix operation used in pairs. The function of the elementary matrix $\mathbf{U_{i,j}}$ in Eqs. (3) and (4) is to add the jth row to the ith row of an adjacency matrix when premultiplying an adjacency matrix, and it is embedded in the subscript. Postmultiplying the transpose of the elementary matrix adds the jth column to the ith column of the adjacency matrix. The new elementary matrix $\mathbf{E_j}$ in Eqs. (3) and (4) is introduced to perform the function of eliminating the jth row of an adjacency matrix when it is premultiplied to the adjacency matrix and eliminating the jth column when its transpose is postmultiplied to the adjacency matrix. Hence, the connectivity is passed on and the number of links reduces. The use of these two matrices results in the EU-elementary matrix operation that performs the required configuration transformation. The matrix operation uses modulu-2 arithmetic, sometimes known as exclusive-or arithmetic [18].

For the transformation of the five-bar configuration [19] into its four-bar configuration, an elementary matrix operation gives

$$\mathbf{A_1} = (\mathbf{E_5}\mathbf{U}_{4,5})\mathbf{A_0}(\mathbf{E_5}\mathbf{U}_{4,5})^{\mathrm{T}}, \tag{3}$$

where

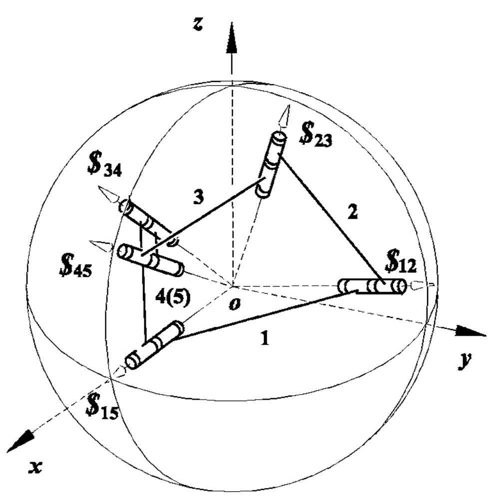

Fig. 2 Configuration state when a five-bar linkage configuration changes to a four-bar linkage configuration

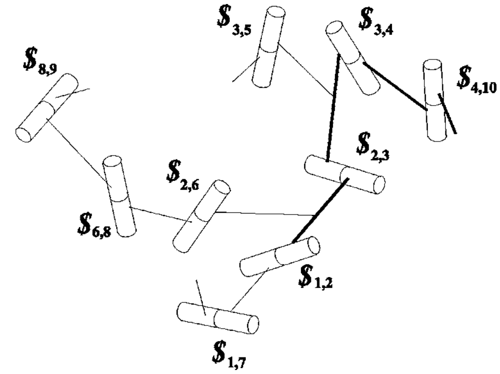

Fig. 3 A spatial mechanism

$$\mathbf{U}_{4,5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{E_5} = [\mathbf{I_4}\ 0] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{4}$$

The above matrix operation in Eq. (3) can be decomposed into two steps. The first step is to apply the U-elementary matrix operation passing the connectivity of link 5 to link 4. This is completed by the following matrix operation:

$$\mathbf{A}_1' = \mathbf{U_{4,5}}\mathbf{A_0}\mathbf{U}_{4,5}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{5}$$

The second step is to apply the E-elementary matrix operation to remove link 5 which is annexed to link 4. This is completed by the following matrix operation:

$$\mathbf{A_1} = \mathbf{E}_5\mathbf{A}_1'\mathbf{E}_5^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \tag{6}$$

Hence, the four-bar configuration is obtained as $\mathbf{A}_1$ from the above decomposed elementary matrix operations. The above illustrative EU-elementary matrix operation can be extended to a complex metamorphic mechanism.

3 Topological-Configuration Transformation of a Spatial Metamorphic Mechanism and its Structural Evolution

A spatial mechanism with nine joints and ten links is shown in Fig. 3. The mechanism was extracted from a cardboard gift carton having a hexahedral form with triangular faces when closed [20].

The precut and creased cardboard profile is initially presented in a developed and flat form. It appears as three squares attached in an "L" formation with four fixing flaps along certain edges of the squares joined with creases which also exist along the diagonals of squares. The six panels from the three squares and four flaps are labeled from 1 to 10 in Fig. 4.

Taking the crease lines as revolute joints and panels as mechanism links [3–5], an equivalent spatial mechanism [21] can be produced and superimposed on the carton as Fig. 4 in which $\$_{i,j}$ stands for a joint axis between panels i and j. The carton is to be erected from its initially flat, cut, and creased form [1] to be a

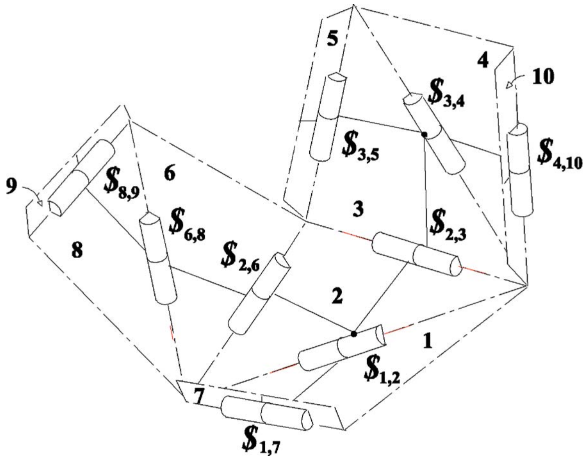

Fig. 4 Equivalent spatial mechanism superimposed on a carton

constrained hexahedron. The topological change resulting from carton folding may produce a number of potentially useful mechanism structures.

This open configuration can be expressed in its adjacency matrix form as follows:

$$\mathbf{A_{0c}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

The manipulation of this equivalent mechanism generates a metamorphic mechanism with four distinct topological configurations, resulting in four different kinematic chains with different mobilities. Transformation relates these topological changes and predicts the new topological configurations. The initial configuration is in Fig. 4, where a general mechanism is there with a mobility of 9. This mobility is determined using the formulas in Ref. [6] based on the Grübler–Kutzbach criterion [22] integrated by Waldron's modification [23] with the order of the screw system [24–27]. The order of the screw system in this case is 6. A metamorphic mechanism can then be produced in the following manipulation by partially erecting the carton when folding panels, 1, 2, 3, 4, and 10 about the creases. When the physical limit is reached [28], panel 10 is attached to panel 1. This operation produces a partially folded carton as in Fig. 5.

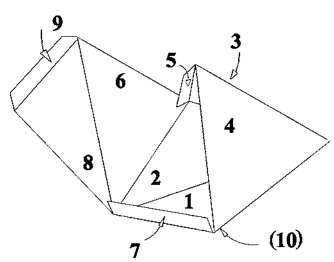

Fig. 5 Partially folded carton

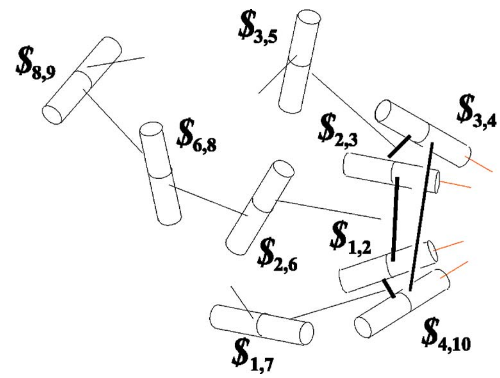

Fig. 6 A spherical four-bar linkage with serial kinematic chains

The metamorphic mechanism can then be formed with a spherical four-bar linkage formed with $\$_{1,2}$, $\$_{2,3}$, $\$_{3,4}$ and $\$_{4,10}$ as highlighted attached with three serial kinematic chains in Fig. 6. The overall mobility is 6 in a similar way to the mobility discussion of the previous configuration.

Applying the EU-elementary matrix operation to the original matrix $\mathbf{A_{0c}}$ as $(\mathbf{E_{10}U_{1,10}})\mathbf{A_{0c}}(\mathbf{E_{10}U_{1,10}})^T$, this configuration transformation can be completed with a resultant adjacency matrix corresponding to the configuration in Fig. 6 as

$$\mathbf{A_{1c}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{8}$$

The next topological configuration appears when the manipulation entails the folding of panel 6 to be attached to panel 5 until the physical limit is reached. Hence, the mechanism changes its configuration to a hexahedral half-structure with mobility 3. This configuration transformation can be represented by an EU-elementary matrix operation, $\mathbf{E_5U_{6,5}A_{1c}}(\mathbf{E_5U_{6,5}})^{\mathrm{T}}$, on the previous adjacency matrix $\mathbf{A_{1c}}$. The adjacency matrix corresponding to this configuration of mobility 3 can hence be obtained from the operation as

$$\mathbf{A_{2c}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{9}$$

The third topological configuration occurs when the manipulation entails the folding of panel 8 to form a hexahedral structure and fixes panel 9 to 4 with joint $\$_{1,7}$ available to operate. The mobility is 1.

This configuration transformation can be represented by an EU-elementary matrix operation on $\mathbf{A_{2c}}$ as $\mathbf{E_8U_{4,8}A_{2c}}(\mathbf{E_8U_{4,8}})^{\mathrm{T}}$. The adjacency matrix corresponding to this configuration is obtained as

$$\mathbf{A}_{3\mathbf{c}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}. \quad (10)$$

The final topological configuration is reached when manipulating joint $\mathbf{\$}_{\mathbf{1},7}$ to attach panel 7 to panel 8; this generates a structure and the carton manipulation is complete. Its adjacency matrix is a result of an EU-elementary matrix operation $\mathbf{E}_7\mathbf{U}_{8,7}\mathbf{A}_{3\mathbf{c}}(\mathbf{E}_7\mathbf{U}_{8,7})^{\mathrm{T}}$ as

$$\mathbf{A}_{4\mathbf{c}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}. \quad (11)$$

The step-by-step operation to transform the initial topological configuration represented by the 10×10 matrix $\mathbf{A}_{\mathbf{0c}}$ to the final topological configuration represented by the 6×6 matrix $\mathbf{A}_{\mathbf{4c}}$ can be summarized by the set of matrix operations

$$\mathbf{A}_{4\mathbf{c}} = (\mathbf{E}_7\mathbf{U}_{8,7}\mathbf{E}_8\mathbf{U}_{4,8}\mathbf{E}_5\mathbf{U}_{6,5}\mathbf{E}_{10}\mathbf{U}_{1,10}) \times \mathbf{A}_{\mathbf{0},\mathbf{c}}(\mathbf{E}_7\mathbf{U}_{8,7}\mathbf{E}_8\mathbf{U}_{4,8}\mathbf{E}_5\mathbf{U}_{6,5}\mathbf{E}_{10}\mathbf{U}_{1,10})^{\mathrm{T}}. \quad (12)$$

4 Conclusions

This paper revealed the intrinsic relationship between topological configurations of metamorphic mechanisms and proposed an analytical way to represent these configurations and facilitate their transformation.

Representing a configuration in the form of topological graph and adjacency matrix, the topological change was modeled by applying matrix transformation on an initial adjacency matrix. This transformation was then decomposed into a number of equivalent EU-elementary matrix operations involving the modulu-2 arithmetic and a new E-elementary matrix for eliminating redundant rows and columns. This leads to a matrix representation of the topological changes of metamorphic mechanisms.

The paper demonstrated the matrix operations in topological changes of a spatial metamorphic mechanism. Configuration changes were described formally as a topological change with a set of matrix operations rigorously laying down procedural information for operators, and are intelligible by machines.

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