Mobility in Metamorphic Mechanisms of Foldable/Erectable Kinds

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Mobility in Metamorphic Mechanisms of Foldable/Erectable Kinds

This paper looks at a class of mechanisms that change structure when erected or folded. The class includes a variety of artefacts and decorative gifts and boxes comprised of flat card creased to enable the folding or unfolding of a structure. Such a structure admits kinematic study in keeping with theory of mechanisms when the creases are treated as hinges joining card and paper panels treated as links. New horizons have been brought up in the use and mechanised manufacture of mechanisms of this kind. Here typical types are described in terms of their fundamental parts and their equivalent mechanisms. Screw system theory is brought into the analysis of mechanisms of these kinds, particularly those containing multiple loops. Different geometry and system combinations are used for the study of mobility and kinematics making use of the result from the equivalent screw systems.

Introduction

A mechanism is most commonly characterised by its function as part of a machine or mechanical arrangement that transforms an input motion or a force into another. If, alternatively, a mechanism has the ability to have its structure transformed from one kind to another then another class of mechanism emerges, one whose primary function may be just to change structure.

Amongst these new developments is a group of devices or arrangements that can be described as mechanisms whose number, the total of all effective links, changes as they move from one configuration to another or a singular condition in geometry occurs that makes it behave differently. We refer to this group of mechanisms as metamorphic. The mechanism may start as an open chain or in a folded (plicated) chain loop to be subsequently erected as a structure.

New interest areas around such mechanisms are growing. For example the study of deployable mechanisms has applications in space technology that requires a highly collapsible and portable mechanism to be carried in a spacecraft and expandable for use either for large antenna structures (Costabile et al., 1996), for ramp assembly (Spence and Sword, 1996) or for the solar array paddle (Kuramasu et al., 1995). New deployable structure has been found in the study of truss structures (Takamatsu and Onoda, 1991). A so called smart fractal structure and mechanisms are recommended in robot manipulators (Shahnipoor, 1993), and sequential logic (Chew and Ho, 1996) was used for the analysis of the mechanisms. A recent study by Pellegrino (1996) focused on one combination of mechanisms and presented a potential application of this kind of mechanisms.

Less obviously related to the above examples is another class typically found in artefacts and fancy gift packs. A list of this kind of application can readily be drawn, including, for example Chinese lanterns, paper folding in Christmas decorations, and card boxes for used in packaging a various of products. Some exotic and innovative forms of the latter represent a technological challenge in producing them by machine. This sets up the need to describe the process in quantifiable kinematic terms. This would also open an avenue for mechanism study leading to innovation in the design of artefacts and packaging.

We usually conceive of mechanisms to be made of ostensibly rigid elements but use relatively flexible card or paper to model and study them. Cundy and Rollett (1951) went into detail on how to make such models. Conversely the artefacts, fancy gift packs and paper foldings that were referred to have not been studied from mechanism theory point of view.

This paper focuses on this new manifestation of a mechanism of a metamorphic kind with particular reference to cut, creased and folded paper and card. Screw theory is used to analyse mobility or structure of these metamorphic mechanisms. The screw theory is also used to classify the mechanisms and to produce the characteristics of the new mechanisms from some classical ones. New families of the mechanism of this kind are presented in the study of a mechanism of this kind. The study presents a coherent correlation between a mechanism and its decorative applications.

Mechanism Models of Artefacts

The application of mechanism theory in the study of mobility in the artefacts, fancy gift packs and paper foldings relies on there being an equivalent mechanism or graph representation. The group of interest is characterised by the fact that they are produced from card or paper that is cut and creased and sometimes prepared with fixing flaps or integral locking devices. They are supplied in a plane flat condition or, very often, folded flat. In general the paper creases act as joints and paper panels act as linkages that allow a mechanism representation (Dai and Rees Jones, 1997a).

A typical example of this kind of metamorphic mechanism is the hexahedron represented Fig. 1. It happens to make a rather nice Christmas tree decoration, whether in coloured card or in textiles, as might be the wont of quilters. In its erect form it appears as a polyhedron with six isosceles triangular faces.

The decoration prior to erection is presented in a flat card form with three squares joined either in *L* form or serially in line. Each square is creased along a diagonal to form the six triangular panels in Fig. 2. In this case there is some assembly to do.

The erection process involves rotating paper panels about the crease lines. The combination of crease lines and panels is analogous to a logical combination of revolute joints and links and presents a mechanism although manifestly in the guise of a decoration. This can be regarded as an equivalent mechanism in Fig. 3(*a*) corresponding to the partially-erect structure in Fig. 3(*b*). Several phases appear in the operation. Each phase of erection is completed by fixing a panel to a flap, perhaps using adhesives such as hot melts. The equivalent metamorphic mechanism changes its link numbers and connectivity in each of the phases. Starting from

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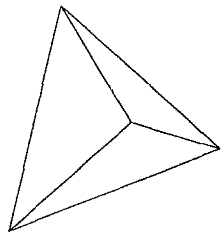

Fig. 1 The hexahedron

Fig. 3(*b*) which is the first phase of the mechanism, rotating paper panels, fixing flap 4′ with panel 1, the two links make a union. The corresponding metamorphic mechanism reduces its link number. In this second phase, the mechanism makes a spherical four-bar linkage with joint (1, 2), (2, 3), (3, 4) and (4, 1) incorporated with another two joints. The rest of the joints are connected and free to move. The next phase appears when fixing flap 3′ to panel 5. The mechanism makes its third phase which becomes an hexahedral shell structure with some free joints. The paper fold ends up in a complete structure. It in fact changes the number of links and connectivity of the mechanism. The mechanism becomes a stiff structure.

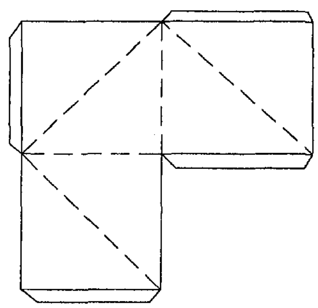

Fig. 2 A flat pre-creased card before its erect form

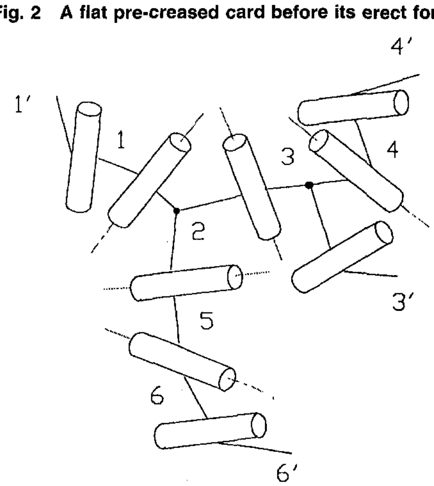

Fig. 3(*a*) An equivalent mechanism

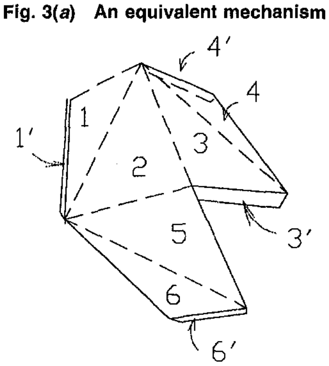

Fig. 3(*b*) Half-erect decoration

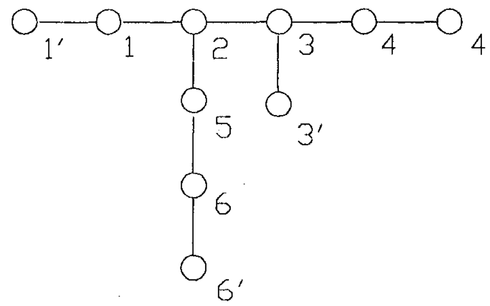

Fig. 4(*a*) Graph representation prior to erect

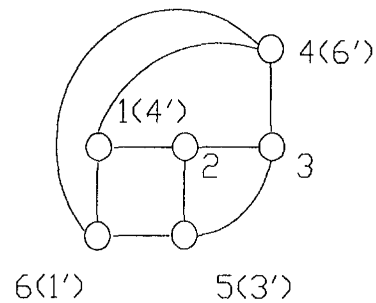

Fig. 4(*b*) Graph representation after erection

A graph representation gives a good illustration of the change of structure of the mechanism. The first and final phase of this metamorphic mechanism are shown in Fig. 4(*a*) and 4(*b*), which give graph representations prior to and after erection. The nodes in the graph represent panels of the decoration and lines represent creases. The graph after erection is given in Fig. 4(*b*), whose corresponding picture is given in Fig. 1. The graph is composed of six nodes representing the six isosceles triangular faces.

Another example usually starts in a folded flat form, reminiscent of a concertina, which generally implies that some panels have already been joined together. This metamorphic mechanism appears in the Chinese lantern. The lantern erects in a near cylindrical form and collapses practically flat. The metamorphosis from flat to erect form is enabled by carefully laid creases in paper defining the edges or joins of a large number of isosceles triangles as in Fig. 5.

Looking closely at one unit of the lantern, the unit structure can be split into two symmetric loops as in Fig. 6. The loop contains

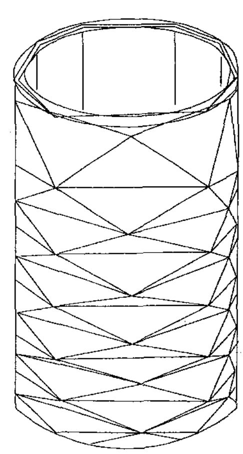

Fig. 5 A Chinese lantern

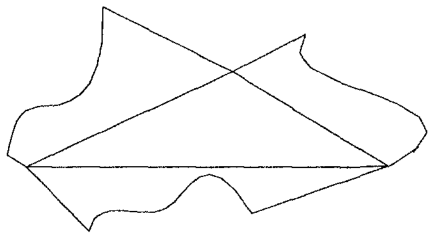

Fig. 6 A loop of half a unit of Chinese lantern

one central panel in the shape of an isosceles triangle defined by three creases and attached with three other isosceles-triangular panels. The creases dominate the folding method and hence the shape of a decoration; they act as axes of revolute joints as in Fig. 7 that allow rotation of two panels about the line to a desirable position. All crease lines are constrained by panels acting as links between revolute joints.

A complete unit of two loops is in Fig. 8 with its equivalent mechanism. The mechanism is a spatial one that is highly expandable and collapsible, as the lantern changes its shape from a folded configuration to a fractal deployable structure (Shahnipoor, 1993) in the form of an erect lantern.

A special feature is associated with this kind of metamorphic mechanism. It happens that they are all revolute joints and the neighbouring axes are coplanar. Hence, the screws (Brand, 1947) that represent the axes of joints are zero-pitch ones, and the scalar products of neighbouring screws are zeros.

Mobility in Metamorphic Mechanisms

The complex nature of the metamorphic mechanism representing these artefacts and fancy packages complicates the study of their mobility, particular in forms of where multiple loops and innovative connections occur. Most of the metamorphic mechanisms do not appear in a conventional way but take a combination of several types of mechanisms. However, the mechanisms can be represented as equivalent screw systems (Hunt, 1978). The mobility analysis would then rely on the study (Hunt, 1967) of these mechanisms in terms of their equivalent screw systems.

The constraint criterion for spatial mechanisms can be stated as a Kutzbach-Grubler criterion (Kutzback, 1937; Hunt, 1959; Suh and Radcliffe, 1978) that is dependent on the type of pair and its constraint. It is stated as

$$F = b(n - 1) - \sum_{i=1}^{m} p_i c_i \qquad (1)$$

where n is the number of links, p_i the number of pairs of type i, and c_i the degree of constraint (degree of freedom lost) at a pair of type i. Scalar b is 6 in the general spatial case and 3 in the planar case. It has been recognised that the above two values of b do not hold for many mechanisms that may be constructed. Rossner (1961) and others introduced a given portion of the mechanism to the appropriate value of b. Boden (1961) split the criterion in two parts and demonstrated that some mechanisms may be dealt with by considering parts of them to obey the planar form and the remainder to obey the spatial form. To a point of generality, Waldron (1966, 1967) and Hunt (1978, 1967) extended the formula by taking account of a general assessment of value of b in terms of the order of an equivalent screw system in a mechanism.

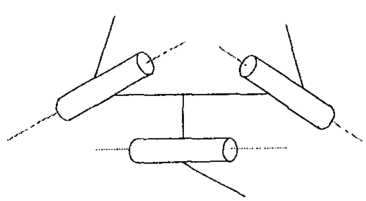

Fig. 7 An equivalent mechanism of a loop in half a unit of the lantern

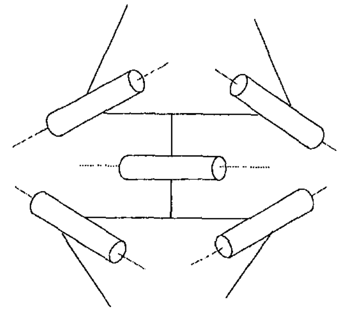

Fig. 8 Mechanism equivalent of a unit of the lantern

Hence the criterion corresponding to the order of a screw system in a mechanism is as follows

$$F = b(n - j - 1) + \sum_{i=1}^{j} f_i \qquad (2)$$

where j is the number of joints and f_i the degree of freedom at the ith joint. The order of a screw system is now incorporated in the formula as b. For an equivalent screw system of order four, the Kutzbach-Grubler criterion holds by taking the coefficient b as four.

If the metamorphic mechanism has revolute joints only, which happens in most cases particularly in most applications with crease lines as joints, the formula evolves

$$F = b(n - j - 1) + j \qquad (3)$$

If a mechanism contains a variety of multiple loop linkages, the criterion holds as long as the equivalent screw system of every loop of the multiple loop linkage has the same order [18], b. When the mechanism contains multiple loop linkages plus open loop linkages, the constraint criterion becomes

$$F = F_c + F_o = b(n - j - 1) + j + j_o \qquad (4)$$

where F_c is the mobility in the close loop, F_o the mobility in the open loop, j_o is number of joints in the open loop. This reflects later in the study of a mechanism that combines parallel and serial mechanisms.

In the mobility analysis of the hexahedral Christmas tree decoration shown in Figs. 1, 2, 3(a) and 3(b), we see that the equivalent mechanism has ten links and nine joints. Six main links, 1, . . . , 6, and five main joints (1, 2), . . . , (5, 6), make a metamorphic mechanism of two loops based on intersecting axes. Since the equivalent screw systems of the two loops have the same order of 3, mobility coefficient b takes 3. The rest four joints (1, 1′), (3, 3′), (4, 4′), (6, 6′) form four open loops, the Eq. (4) can be used as follows

$$F = 3(6 - 5 - 1) + 5 + 4 = 9 \qquad (5)$$

The mechanism changes its phase when fixing link 4′ to link 1. As discussed in previous section, it becomes a spherical four-bar linkage incorporated with another two links plus three open loops in this second phase. The mobility is

$$F = 3(6 - 6 - 1) + 6 + 3 = 6 \qquad (6)$$

The next motion changes the metamorphic mechanism to another phase and mobility reduces to 3. Table 1 shows the progression from each phase to the next and the resulting mobility.

Table 1 **Mobility changes with phases of a hexahedral equivalent mechanism**

Phase	Desirable movement	Mechanism appeared	Mobility
1	Free movement	General metamorphic mechanism	9
2	Fix link 4′ to 1	Spherical 4-bar incorporated	6
3	Fix link 3′ to 5	Hexahedral half-structure	3
4	Fix link 6′ to 4	Structure with one free joint	1
5	Fix link 1′ to 6	Structure	0

Further, a different order of fixing movement produces different phases of the metamorphic mechanism but same mobilitiy in each phase. If the second phase above is to fix flap 3′ to panel 5, the three links of 2, 3, 5 become a triangular pyramid structure with rest joints free to move. The mobility is 6 from Eq. (4). Hence Table 2 can be given.

When the decoration reaches its final phase, the number of links reduces to 6 from 10, there is no relative motion between links and the mechanism becomes a complete structure.

Now we consider the mobility of a Chinese lantern, in Fig. 5. Referring to an equivalent mechanism in a loop of half a unit of the lantern in Fig. 7, the screw axes to construct three joints are on the same plane and hence form a three-system. The mobility is three as

$$F = 3(4 - 3 - 1) + 3 = 3 \tag{7}$$

Consider a complete unit in Fig. 8 that takes the stack of two loops, the two equivalent screw systems for two loops have the same order of three, the mobility is given as follows

$$F = 3(6 - 5 - 1) + 5 = 5 \tag{8}$$

The study is then extended to one layer of the lantern and considers different configurations. It is equivalent to a parallel mechanism (Rees Jones, 1987). The mobility varies with the number of units in the mechanism. The equivalent screw system for each unit is the same and has the same order.

The mobility of a layer comprising three units, represented by the mechanism in Fig. 9(*a*), is

$$F = 3(4 \times 3 - 5 \times 3 - 1) + 5 \times 3 = 3 \tag{9}$$

Similarly, the mobility of a layer of four units in Fig. 9(*b*) is 5. The mobility of a single-layer lantern constructed with five units is 7. A pattern of the mobility of such a mechanism with one layer and multiple units can hence be given. Start from mobility of three for a lantern of one layer with three units, each incremental unit adds mobility two to the existing mobility. A formula can be given

$$F_1 = 3 + 2(u - 3) \tag{10}$$

Table 2 **Change of the order of movement of the hexahedral equivalent mechanism**

Phase	Desirable movement	Mechanism appeared	Mobility
1	Free movement	General metamorphic mechanism	9
2	Fix link 3′ to 5	Pyramid with free joints	6
3	Fix link 1′ to 6	Hexahedral half-structure	3
4	Fix link 4′ to 1	Structure with one free joint	1
5	Fix link 6′ to 4	Structure	0

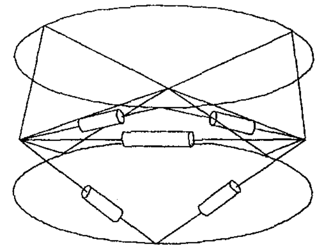

Fig. 9(*a*) A layer with three units

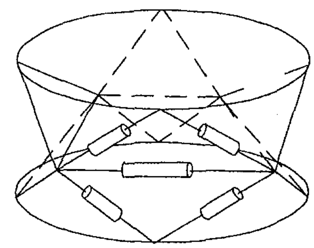

Fig. 9(*b*) A layer with four units

where u stands for the number of units of a layer and starts from 3 in this simplified equation. A lantern constructed in multiple layers is a mechanism connecting individual parallel mechanisms in serial. A lantern with two layers is shown in Fig. 10(*a*) and three layers in Fig. 10(*b*).

A general mobility formula for a lantern with multiple units and multiple layers is given as follows

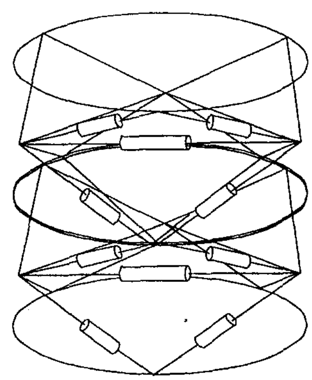

Fig. 10(*a*) A lantern mechanism with two layers

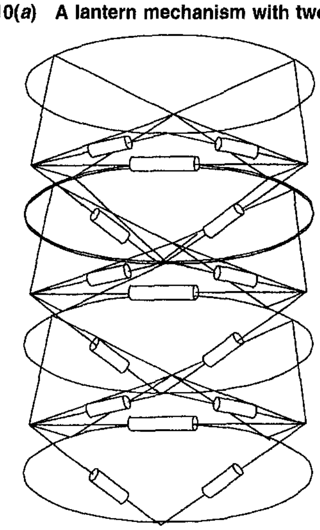

Fig. 10(*b*) A lantern mechanism with three layers

Table 3 Mobility of Chinese lantern equivalent mechanism with different configurations

No. of units in a layer / No. of layers	3	4	5	6	...
1	3	5	7	9	Odd number
2	9	14	19	24	$2F_1+u$
3	15	23	31	39	$3F_1+2u$
...	$lF_1+(l-1)u$	...	...	...	... ...

$$F = \sum_{i=1}^{l} (3(4u_i - 5u_i - 1) + 5u_i) + \sum_{i=1}^{l-1} u_i \qquad (11)$$

where l is the number of layers, u the number of units in a layer. The mobility of different layout of lanterns is shown in Table 3.

A pattern can be seen for the mobility of this kind of mechanism in the last column of Table 3.

The mobility has a clear pattern in terms of the number of layers in the mechanism. Each incremental layer adds mobility to the existing one by the mobility of one layer and extra mobility the same number as the number of units being added. A simplified form of Eq. (11) for multiple layer lanterns is

$$F_m = lF_1 + (l-1)u \qquad (12)$$

Metamorphic Mechanism Based on Parallel Axes

A commonly found metamorphic mechanism is one in which parallel axes dominate the possible motion. It finds its application as a cardboard box container with a crash-lock base in Fig. 11.

The four main panel sections present four links while the four parallel creases present four revolute joints to form a main structure of the box. This equivalent linkage of the main structure is attached with a base formed by other four panels and six creases which form two extra loops. The mechanism representation of the box with its base is presented in Fig. 12. The ten revolute joints form a spatial mechanism.

The corresponding graph representation is presented in Fig. 13. The mechanism has three loops that are equivalent to three screw systems of order three constructed by the axes of joints (Dai and Rees Jones, 1997b). Applying the extended Kutzbach-Grubler mobility Eq. (2), the mobility is

$$F = 3(8 - 10 - 1) + 10 = 1 \qquad (13)$$

A physical interpretation follows. The three loops are formed by a four-bar linkage, two folding sub-mechanisms acting as bulldog clips in Fig. 14 at two edges of the four-bar linkage. The mobility for each sub-mechanism is one. Hence, the mobility for the whole should not be over one.

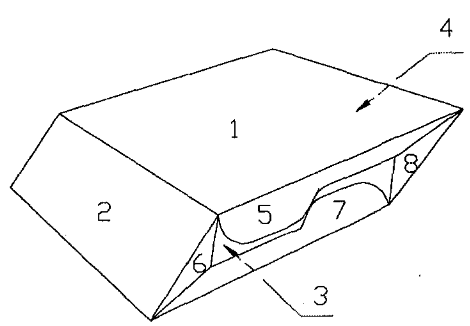

Fig. 11 A card box with a crash-lock base

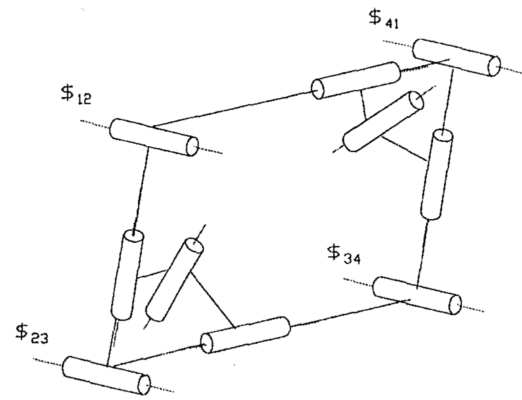

Fig. 12 A special mechanism equivalent to the box

Equations can be evolved for each of the loops. The main loop equation in Fig. 12 can be written as the sum of the screws representing the joint as

$$\$_{12} + \$_{23} + \$_{34} + \$_{41} = 0 \qquad (14)$$

The sub-loops are shown as folding sub-mechanisms similar to bulldog clip devices in Fig. 14. The equation for that is given as

$$\theta_{67}\$_{67} = \theta_{26}\$_{26} + \theta_{23}\$_{23} + \theta_{37}\$_{37} \qquad (15)$$

which can be written in a matrix form

$$\theta_{67}\$_{67} = [\$_{26} \quad \$_{23} \quad \$_{37}] \begin{bmatrix} \theta_{26} \\ \theta_{23} \\ \theta_{37} \end{bmatrix} \qquad (16)$$

The mechanism has two phases, a mechanism phase, and a structure phase where the mechanism changes its connectivity. It occurs when the sub-mechanisms reach a singular position, the number of links is reduced by annexing other links and the joints reach their limits in the physical constraints. The mechanism becomes over-constrained with no relative motion. The box becomes a structure as in Fig. 15. The corresponding graph is given in Fig. 16.

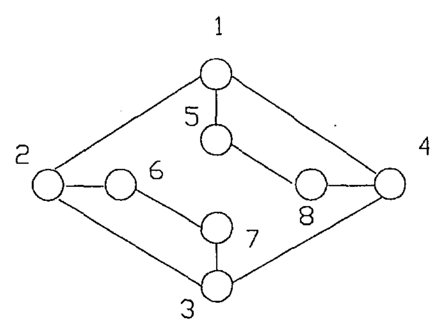

Fig. 13 A graph representation

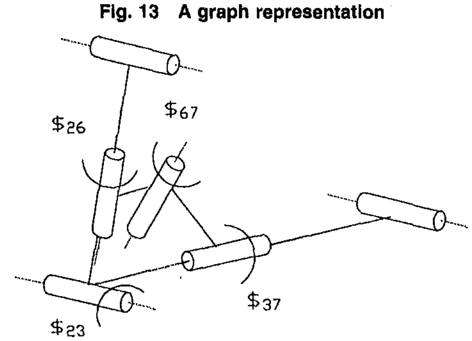

Fig. 14 A sub-loop of the mechanism

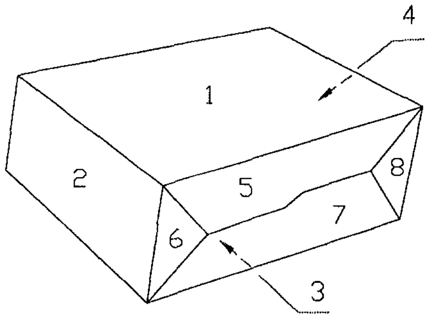

Fig. 15 The erect box

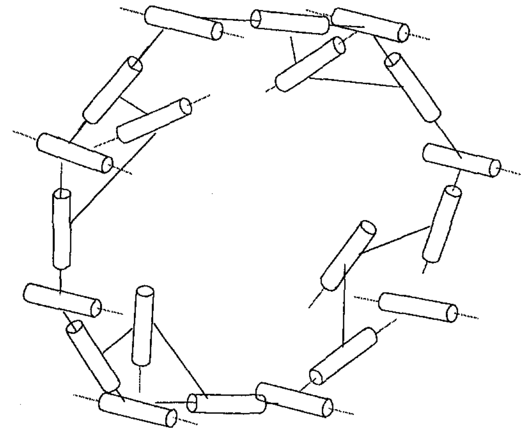

Fig. 18 Equivalent mechanism of an octagonal box

Family of the Metamorphic Mechanisms Based on Parallel Axes

The above metamorphic mechanism has many variations. A typical variation is noticed in a hexagonal card box. The equivalent mechanism is shown in Fig. 17. Four loops of linkages are spotted comprising four screw systems of order three. The extended mobility equation is used to produce the mobility of the mechanism as

$$F = 3(12 - 15 - 1) + 15 = 3 \tag{17}$$

The three degrees of freedom are given by the centre joints of the three folding devices which are acting as active joints.

The extended mobility equation is used to produce the mobility of the mechanism as

$$F = 3(12 - 15 - 1) + 15 = 3 \tag{18}$$

The three degrees of freedom are given by the centre joints of the three folding devices which are acting as active joints. The kinematics equation of the main loop of the six parallel axes is given as

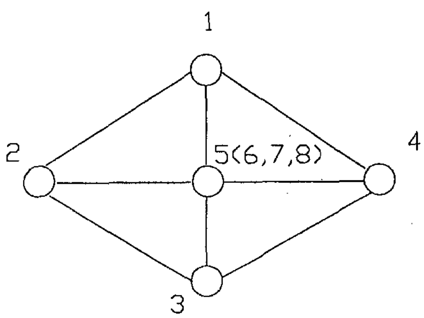

Fig. 16 A corresponding graph

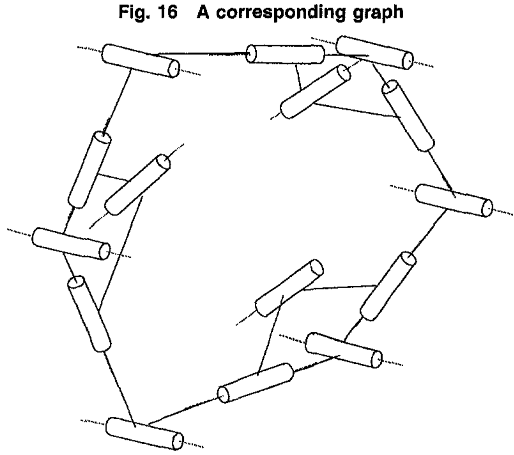

Fig. 17 An equivalent mechanism of a hexagonal card box

$$\sum_{i=1}^{6} \$_i = 0. \tag{19}$$

The three sub-loops of folding sub-mechanisms are given as

$$\theta_j \$_j = \mathbf{B}_j \Theta_j, \quad j = 1, 2, 3, \tag{20}$$

where $\mathbf{B}_j$ is a matrix comprising screw representations of axes of joints of the jth folding sub-mechanism. Θ_j is a vector comprising angular displacements of the jth folding sub-mechanism. In the study of this class of mechanisms, $\mathbf{B}_j$ is 6×3 and Θ_j is a three-element vector.

When the metamorphic mechanism changes configuration and phases, the number of links changes when their sub-mechanisms reach their end position. In this position, link number reduces, the mechanism becomes rigid with no relative motion. A further variation is an octagonal decorative card box. The equivalent mechanism is shown in Fig. 18.

Five screw systems that form five loops of linkage are presented as axes of joints. The screw systems have the same order of three. Hence the mobility is

$$F = 3(16 - 20 - 1) + 20 = 5 \tag{21}$$

A general formula for the family of this kind of metamorphic mechanisms can be given as

$$\begin{aligned} F &= b(2n - (2n + c) - 1) + (2n + c) \\ &= b(-c - 1) + 2n + c \end{aligned} \tag{22}$$

where, b is the order of an equivalent screw system, n the number of parallel axes equivalent to the edges of the tubloidal card box, c the number of folding sub-mechanisms corresponding to the number of diagonal creases on the base of the box, and equal to $n/2$. The equation hence becomes

$$F = b\left(-\frac{n}{2} - 1\right) + 2n + \frac{n}{2} = \frac{n}{2}(5 - b) - b \tag{23}$$

Since the order of screw systems for this family of metamorphic mechanisms is always three, the equation is simplified as follows

$$F = n - 3 \tag{24}$$

Hence, this family of the mechanism can be extended to many variations with the above mobility formula.

The kinematics equations are

$$\sum_{i=1}^{n} \$_i = 0, \tag{25}$$

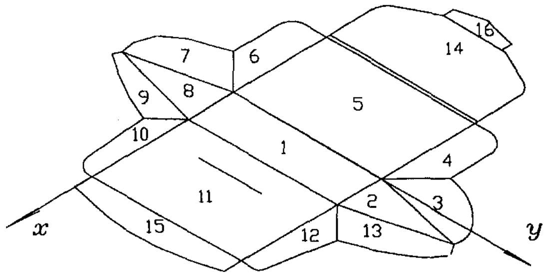

Fig. 19 A flattened configuration of a card box

$$\theta_j \$_j = \mathbf{B}_j \Theta_j, \quad j = 1, \ldots, m. \tag{26}$$

where m is the number of folding sub-mechanisms. When the mechanism becomes a structure in one of its phases the mechanism becomes symmetric and erect. All folding sub-mechanisms in sub-loops are coplanar. The links of all folding sub-mechanisms are annexing to make a union and become rigid.

There is a comparison between this kind of mechanism and the planar mechanism. Removing from the above mechanisms the additional spatial parts of those pairs of folding sub-mechanisms, the mechanisms become planar ones of four-bar, six-bar and eight-bar linkage. The mobility from these planar mechanisms is the same as obtained above. In such a vein, the closed folding sub-linkages added in the above arrangement do not change the mobility of an initial linkage. The effect of it is to limit the linkage movement and act as a folding device for a mechanism to be expandable and collapsible. Thus the parallel axes in this kind of mechanism determine the mobility of the mechanism. The mechanism is hence called a metamorphic mechanism based on parallel axes.

Metamorphic Mechanisms Based on Intersecting Axes

A typical intersecting axis based metamorphic mechanism is the hexahedron and the one in the form of Chinese lantern. More cases can be found in Christmas decorations and in some fancy and novel card boxes, in particular when the folding process is to wrap a content.

A typical illustration of the application of this kind of mechanism is a boy-scout tent card box. The flattened and operating configurations of the card box are shown in Fig. 19. The crease lines are arranged with different shapes of panels with no clear distinct characteristics of loops and mechanisms. The crease lines are treated as axes of screws indicated as line vector **D** (Dai and Rees Jones, 1997c). The half-erect box with these screws is shown

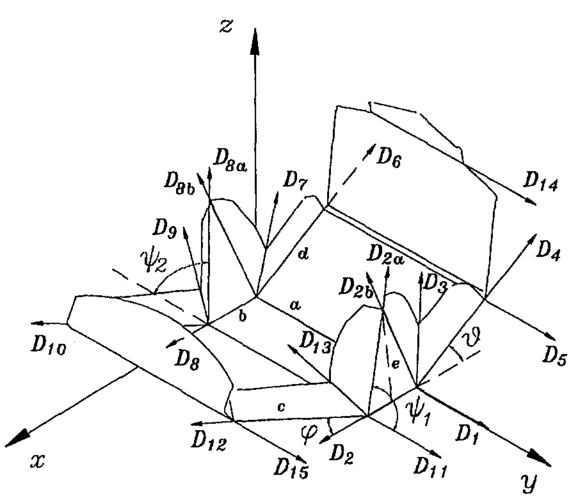

Fig. 20 Half-erect card box with screw axes

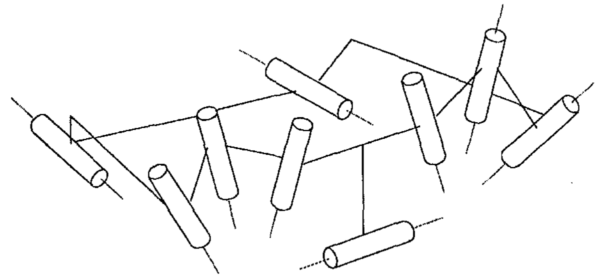

Fig. 21 An equivalent mechanism of half of the box

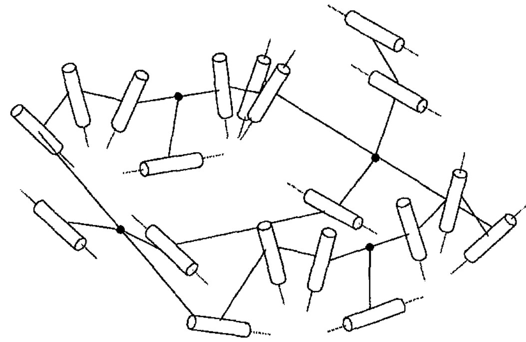

Fig. 22 Equivalent mechanism of a whole box

in Fig. 20, where line vector **D** is given to describe a joint axis as a finite twist (Dai and Holland and Kerr, 1995, 1996).

Starting from the right half of the carton on the symmetry line for panel 1–5 and 11–13, an equivalent mechanism is drawn in Fig. 21. Two loops of the mechanism can be found which construct two folding sub-mechanisms. Equivalently, and two screw systems of order three are found.

Hence the mobility taking account of two screw systems of order three can be given as

$$F = 3(8 - 9 - 1) + 9 = 3 \tag{27}$$

Two folding sub-mechanisms and one joint at the base contribute the mobility. The kinematics equation can be given by the two folding sub-mechanisms as follows

$$\theta_j D_j = \mathbf{B}_j \Theta_j, \quad j = 1, 2. \tag{28}$$

Taking a whole box, the equivalent mechanism is presented in Fig. 22. Four closed loops with four folding sub-mechanisms are found. The screw systems that are equivalent to the loops have the same order. Since there are two open loops with three joints, from Eq. (4), the mobility of the complete mechanism can be calculated as

$$F = (3(13 - 16 - 1) + 16) + 3 = 7 \tag{29}$$

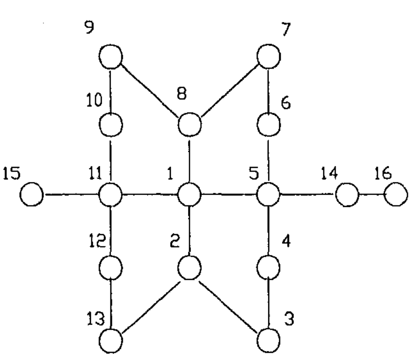

Fig. 23 A corresponding graph representation

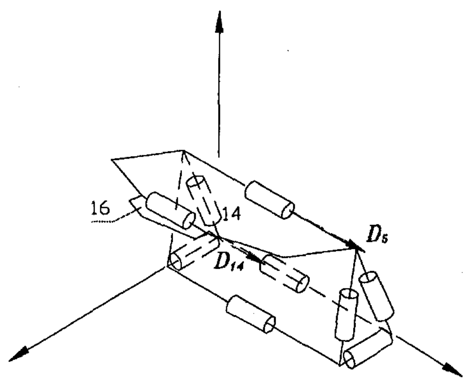

Fig. 24(*a*) A phase of mobility two

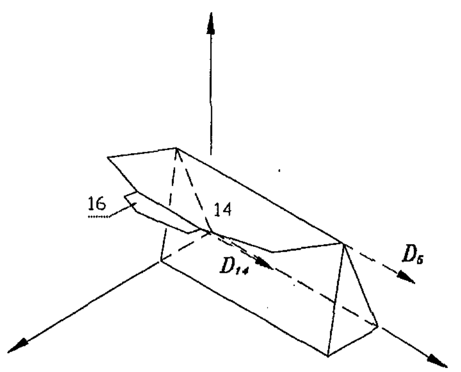

Fig. 24(*b*) Corresponding configuration of the box

The mobility hence is seven with the contribution of three from open linkages. A corresponding graph representation is given in Fig. 23.

The metamorphic mechanism changes its number of links and consequently changes the structure of the mechanism. One phase of the mechanism is shown in Fig. 24(*a*). The corresponding configuration of the box is shown in Fig. 24(*b*). Two degrees of freedom are left in this phase.

The last phase of mechanism is given when number of links reduces to 5 as the graph representation in Fig. 25. The mechanism becomes a structure.

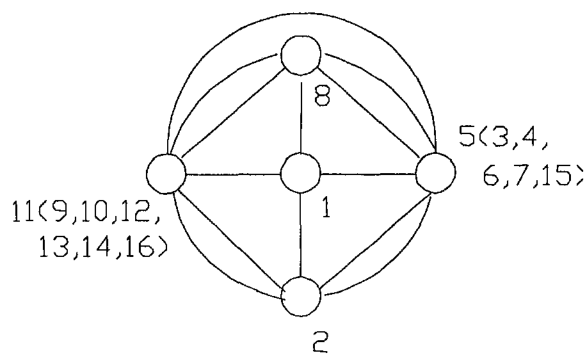

Fig. 25 A graph representation of the structure configuration of the mechanism

Conclusion

A new class of mechanism has been discussed which has in particular the ability to change its structure by changing link numbers and connectivity. An application of such a metamorphic mechanism and its extensions in the sense of being highly expandable and collapsible have been discussed in the context of a variety of decorations and card boxes. The mechanism and its extensions are used for folding and manipulating process in a way equivalent to a mechanism device. The mobility and kinematics of the mechanism, that is equivalent to a decorative box, were discussed in terms of the order of its equivalent screw system. Two typical metamorphic mechanisms based on parallel and intersecting axes were discussed being used in most decorations in their manipulating and folding operation. The discussions and approach of study present a new area for kinematics analysis of metamorphic mechanisms in application to decorative box design and manipulating process, and provide a way in the study of metamorphic mechanisms.

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