

Huijuan Feng<sup>a</sup>, Wujie Shi<sup>a</sup>, Pino Trogu<sup>b</sup>, and Jian S. Dai<sup>a</sup>, *Fellow, IEEE*

# KINEMATIC MODELING OF A FLAT-FOLDABLE AUXETIC METAMATERIAL

THE 6TH INTERNATIONAL CONFERENCE ON RECONFIGURABLE  
MECHANISMS AND ROBOTS (REMAR 2024) – CHICAGO, U.S.A. JUNE 23–26, 2024  
MONDAY, JUNE 24, 2024 – 2:30 PM

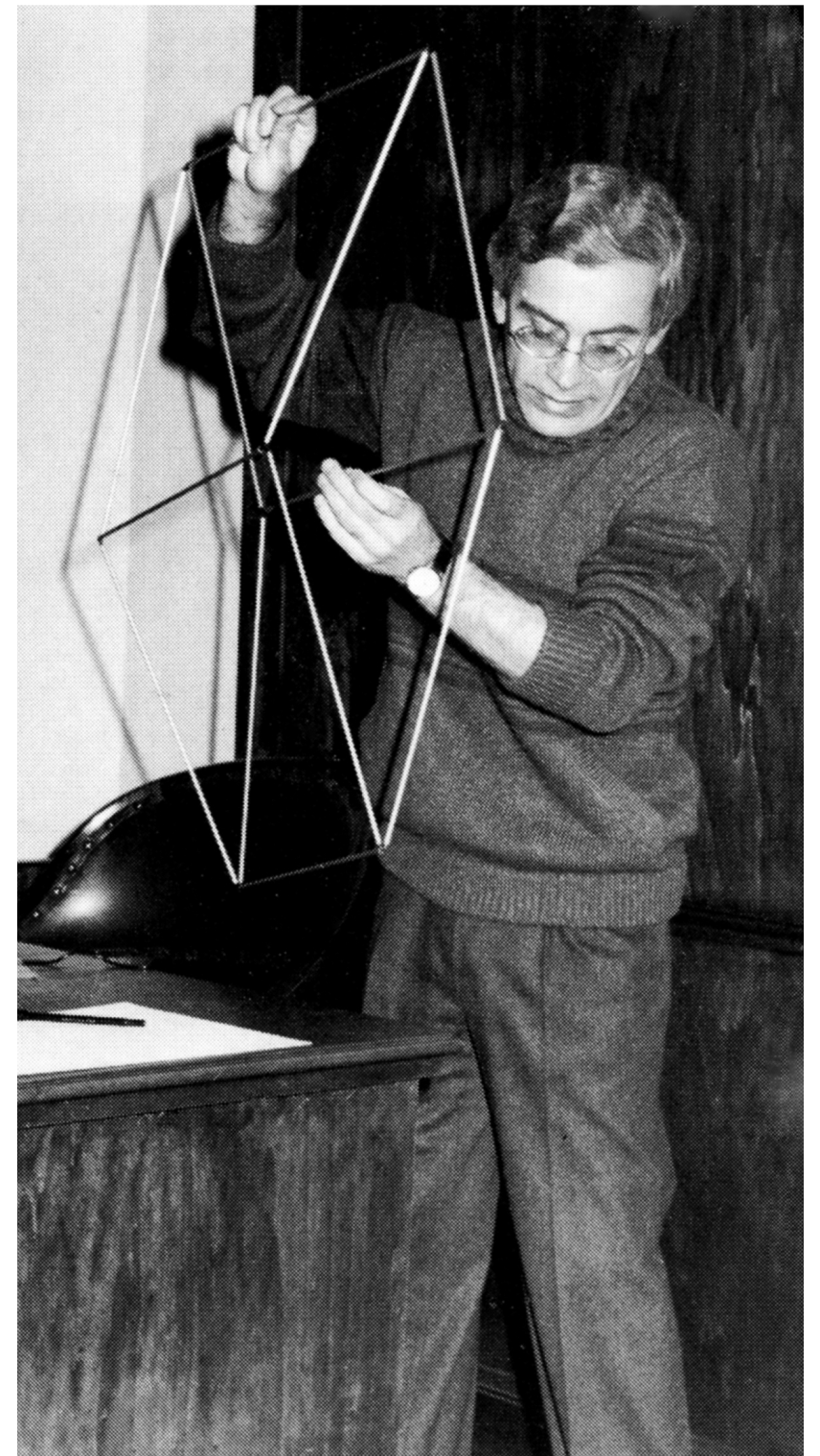
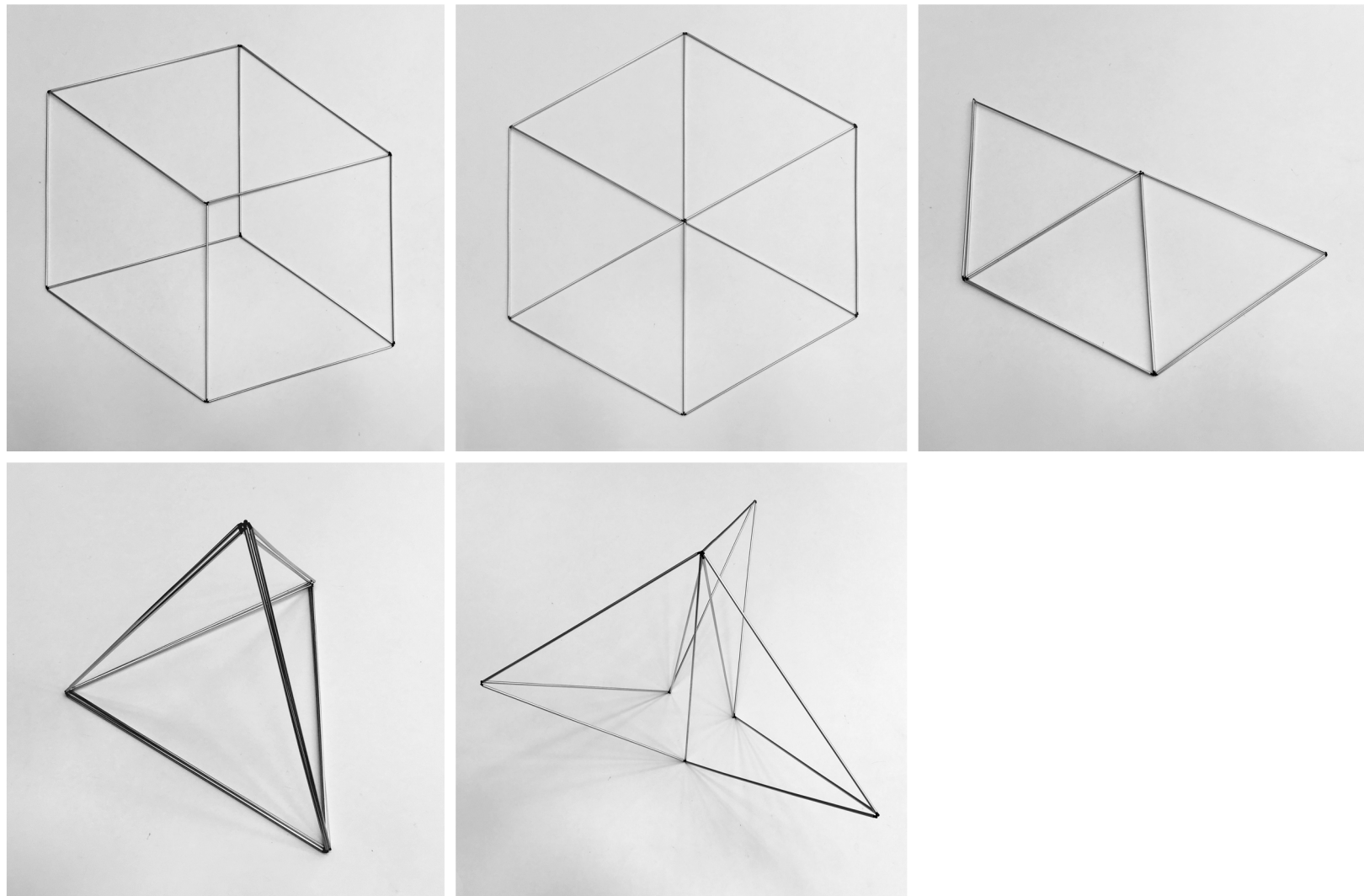
<sup>a</sup> Southern University of Science and Technology, Shenzhen, China

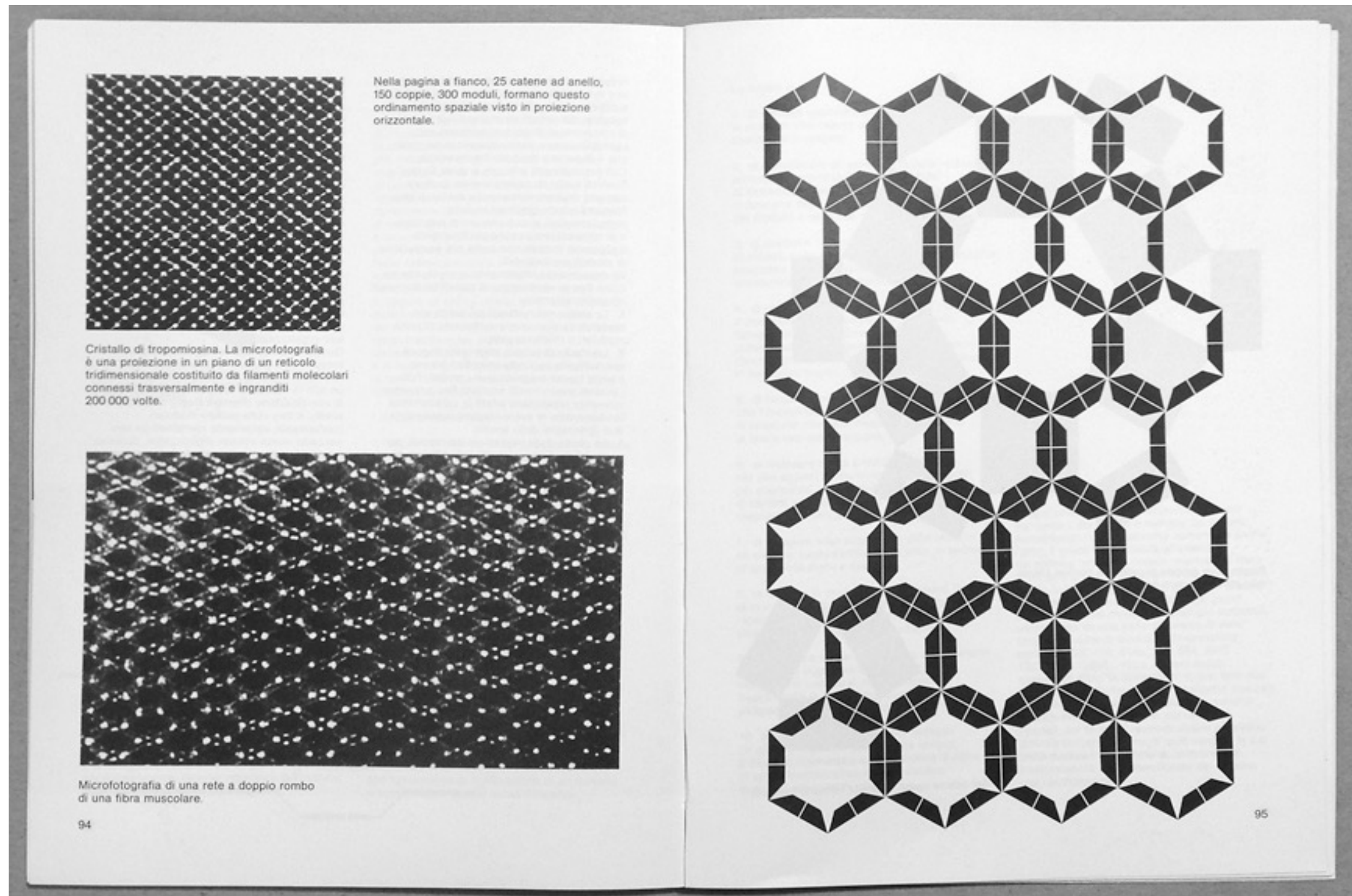
<sup>b</sup> San Francisco State University, U.S.A.

Corresponding author – email: trogu@sfsu.edu (P.Trogu)

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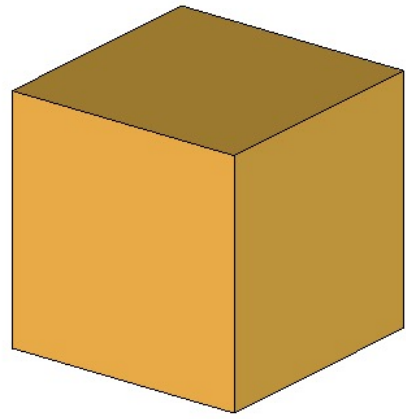




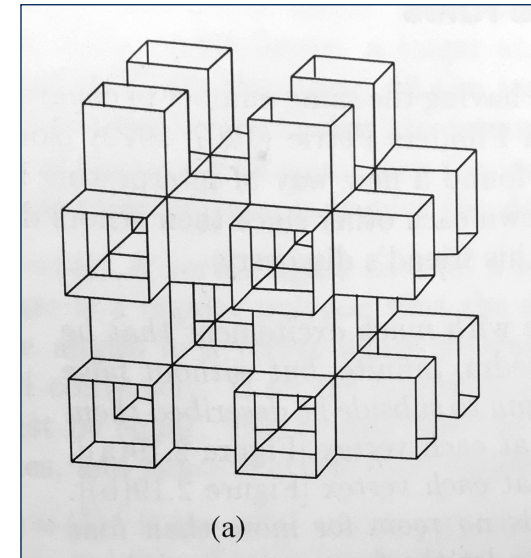
(G. Scarpa, Models of Rotational Geometry, 1978)

TWO  
TRANSFORMABLE SHAPES:  
“FIGURE TRASFORMABILI”  
1996

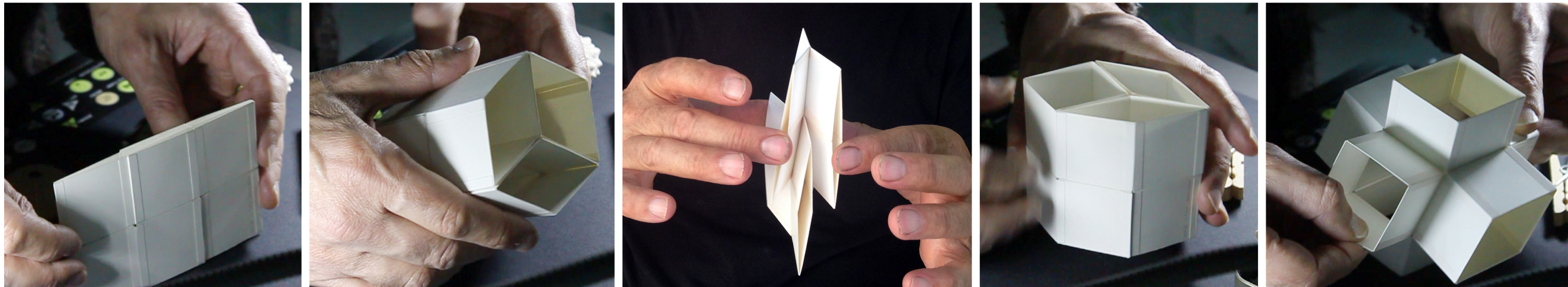




**J. F. PETRIE**  
**REGULAR HONEYCOMB, 1926**



**G. SCARPA, 1996**



**B. OVERVELDE *ET AL*, 2016**

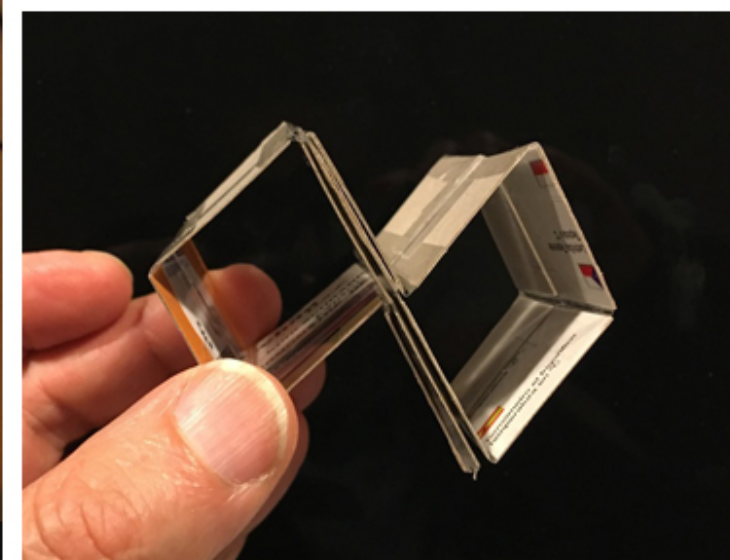
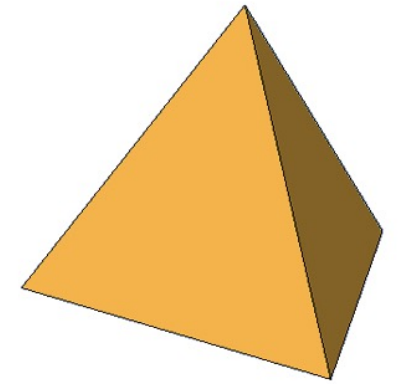
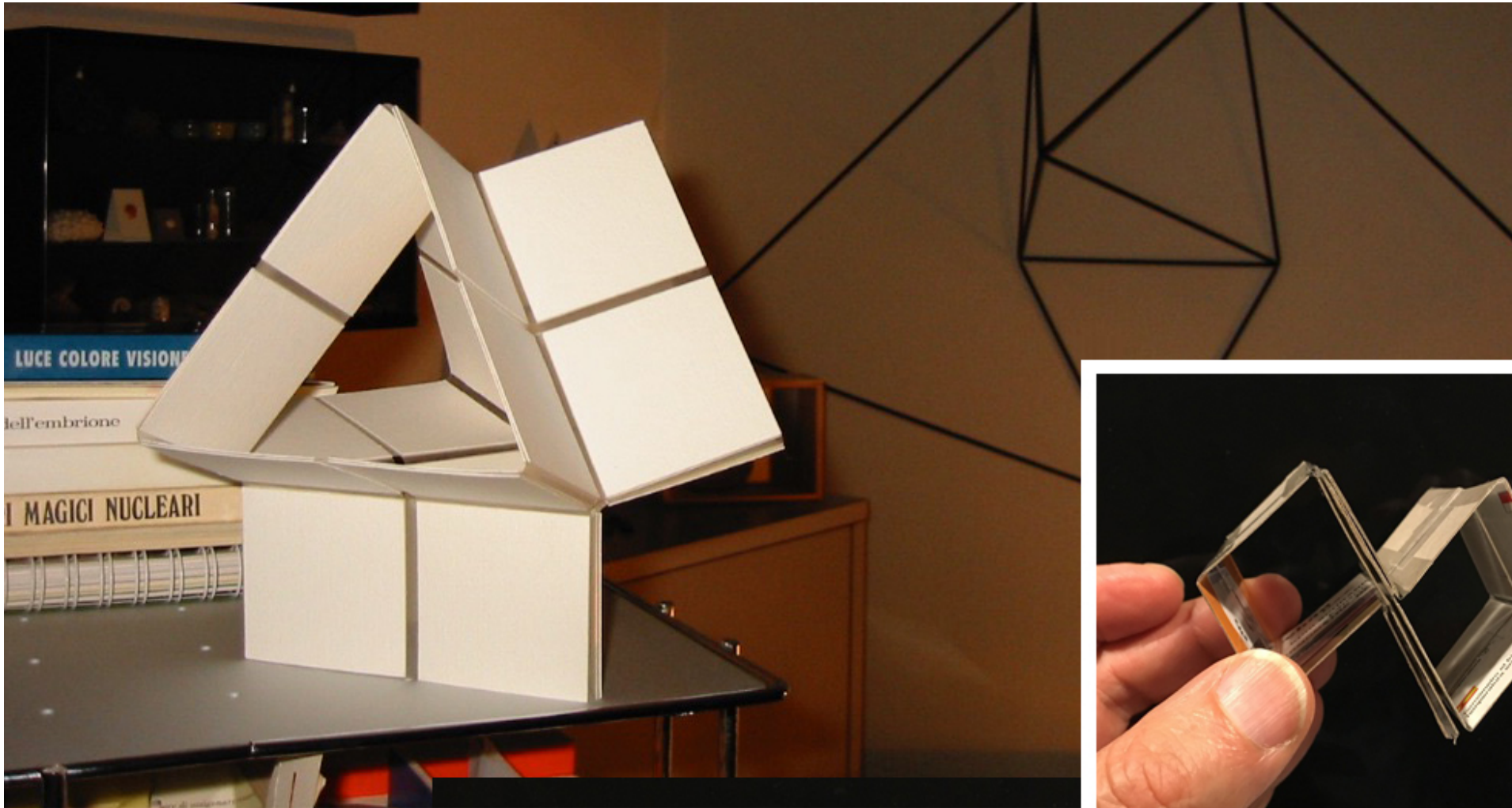


*“I hope you like the transformable shape you will find in the package. But it was something else that I wanted to send you, a paper object that I built in three different versions but which still has more than one problem to be resolved.”*

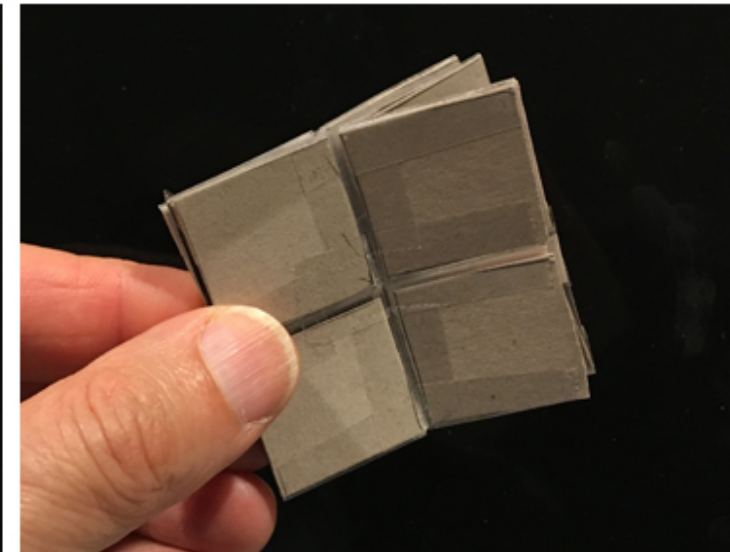
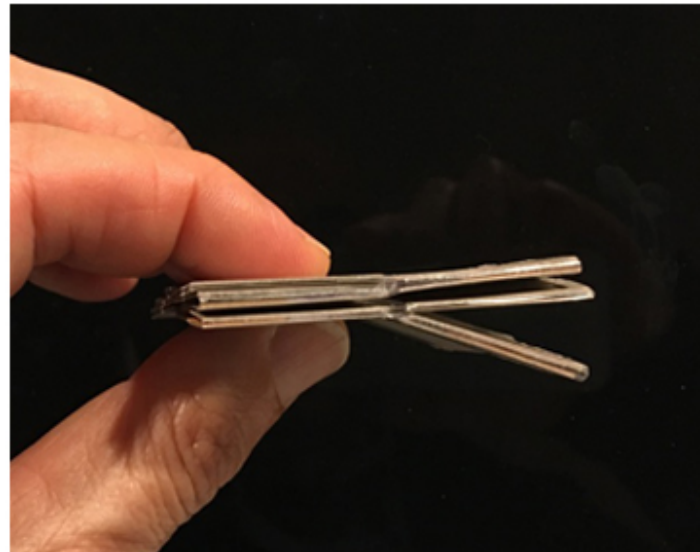
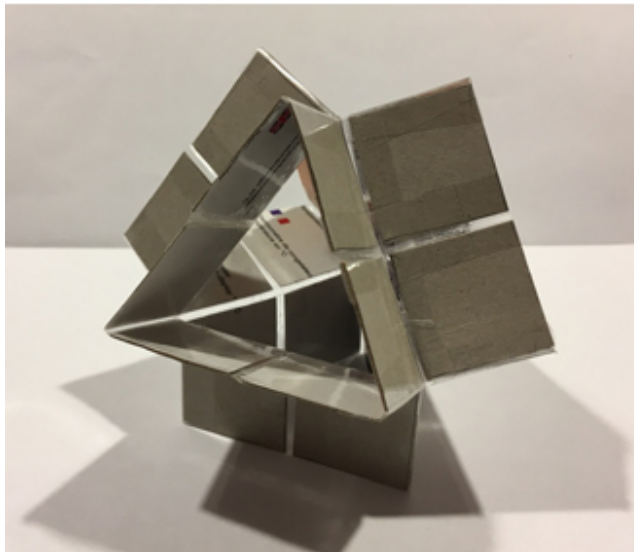
Letter from Giorgio Scarpa to Pino Trogu  
July 28, 1996



## G. SCARPA – SPLIT EXTRUSION TETRAHEDRON – 1996

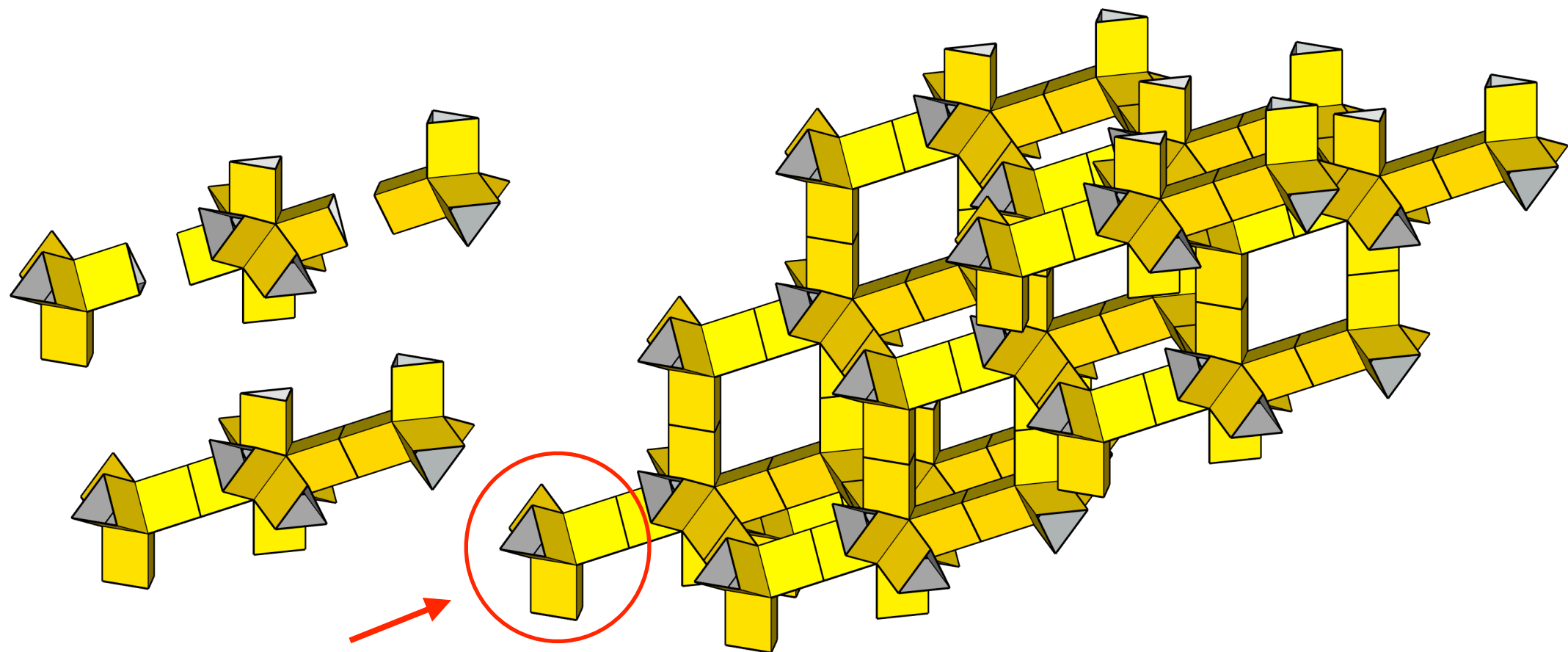
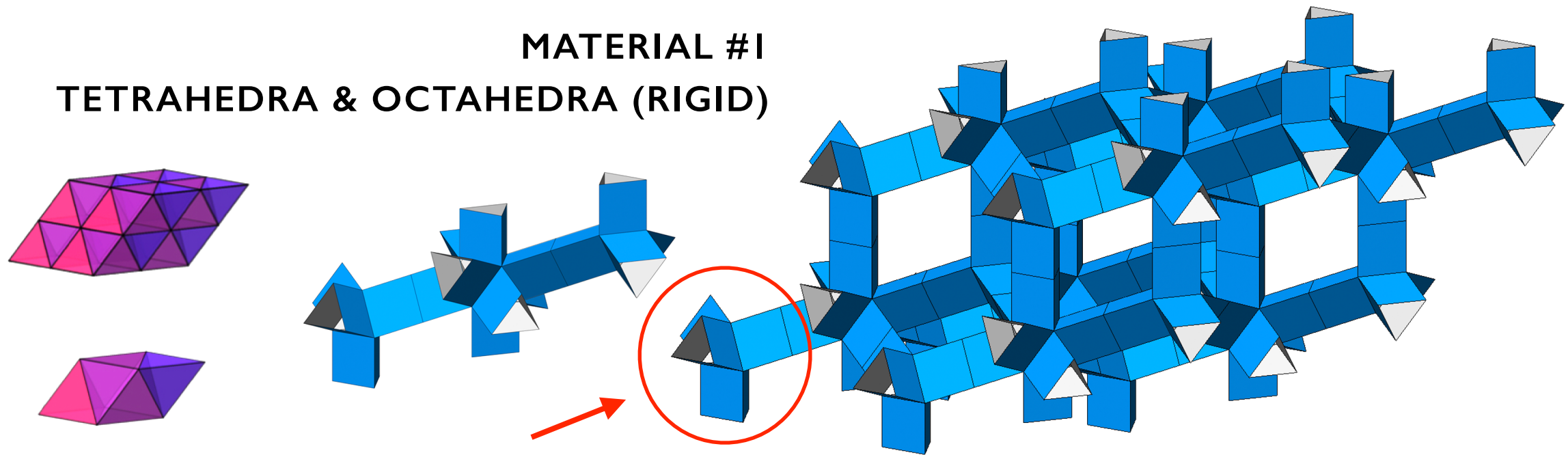


(REPLICA)  
P. TROGU  
2017



**METAMATERIALS:**  
**RIGID,**  
**NON-RIGIDLY FOLDABLE,**  
**& RIGIDLY FOLDABLE.**

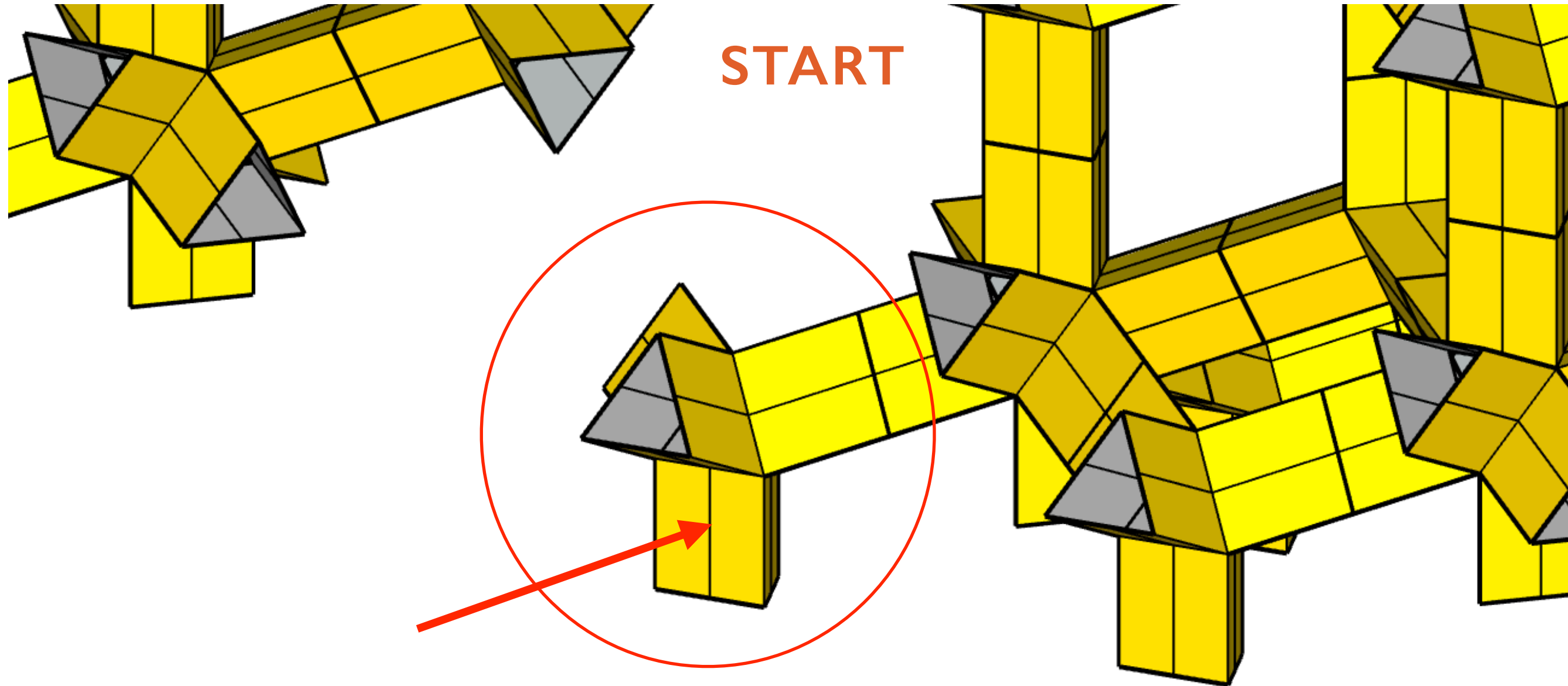
**MATERIAL #1**  
**TETRAHEDRA & OCTAHEDRA (RIGID)**



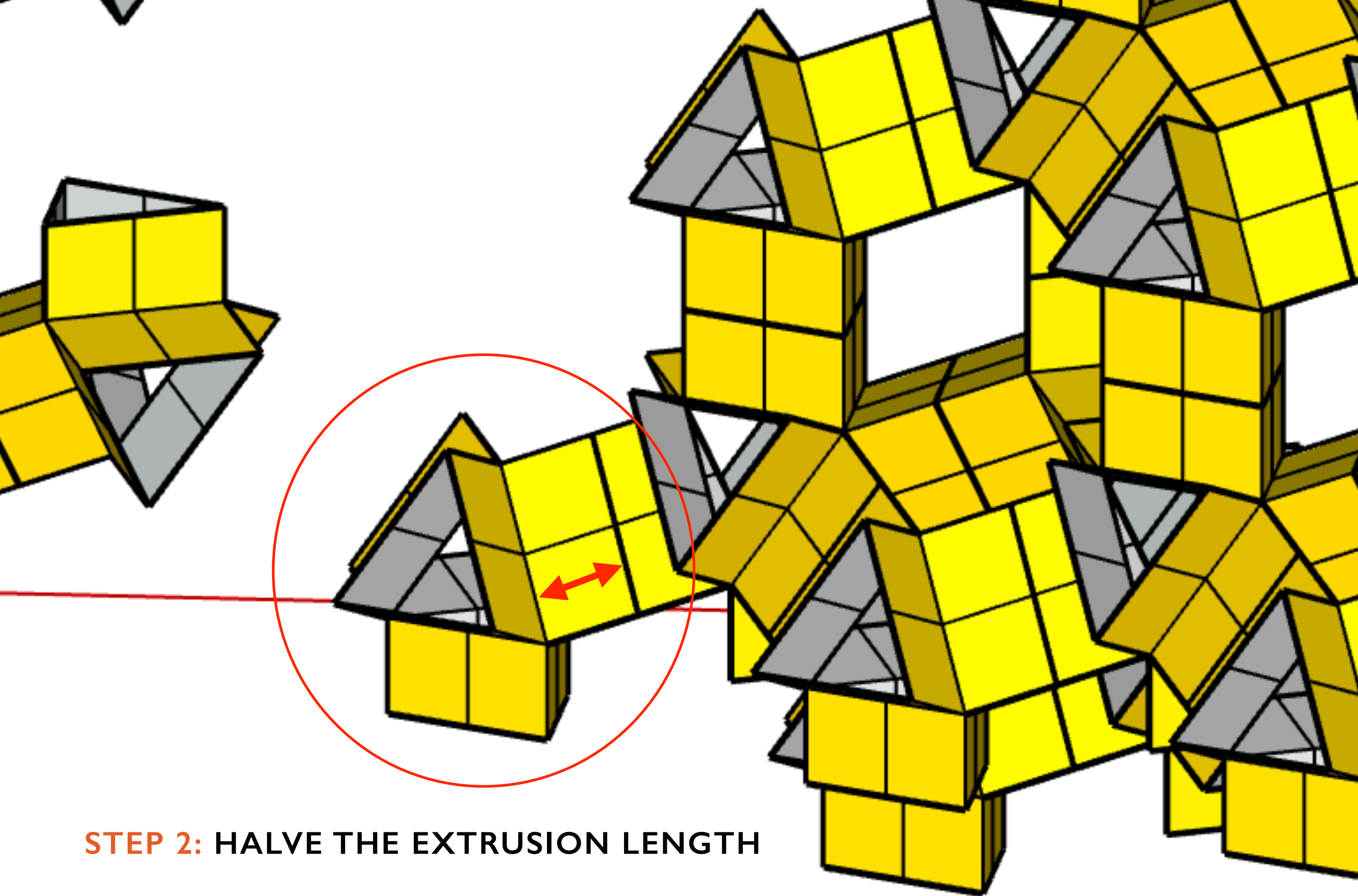


MATERIAL #1  
TETRAHEDRA & OCTAHEDRA (FLAT-FOLDABLE)

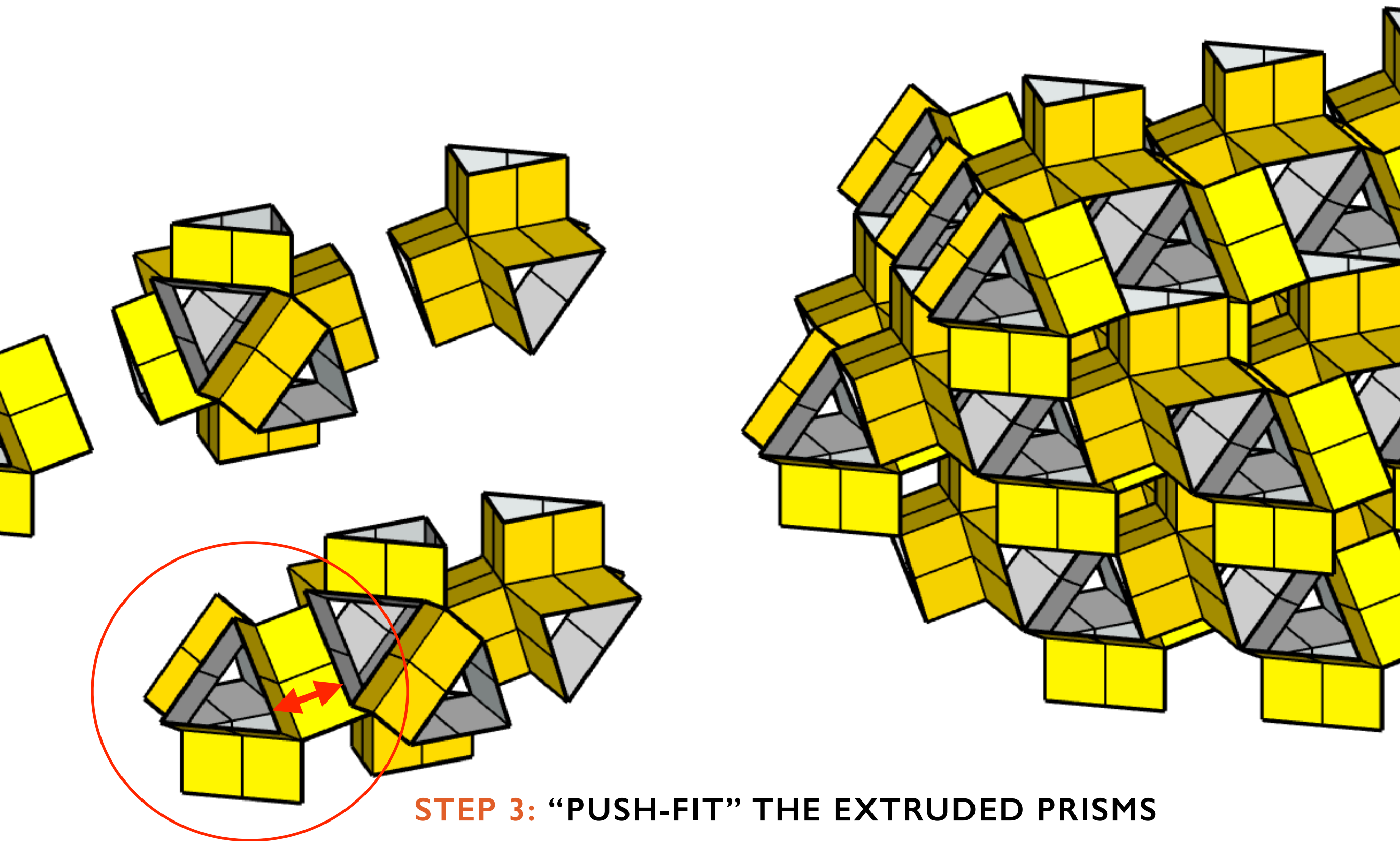
START



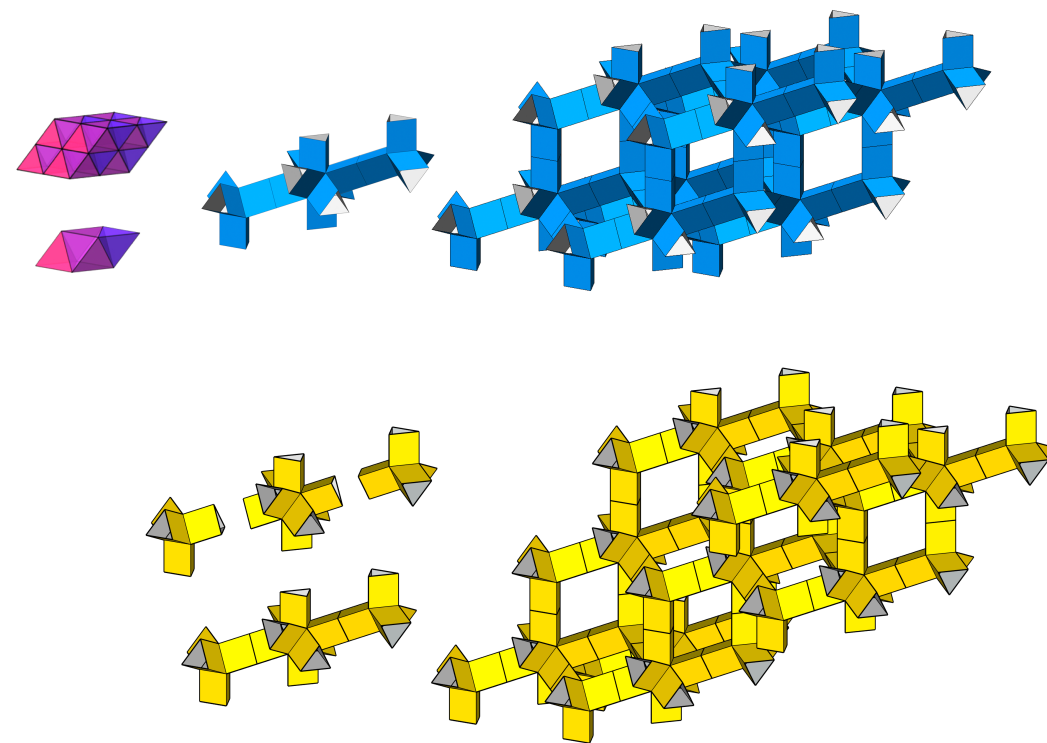
**STEP 1: BISECT EXTRUDED PRISM WALLS**



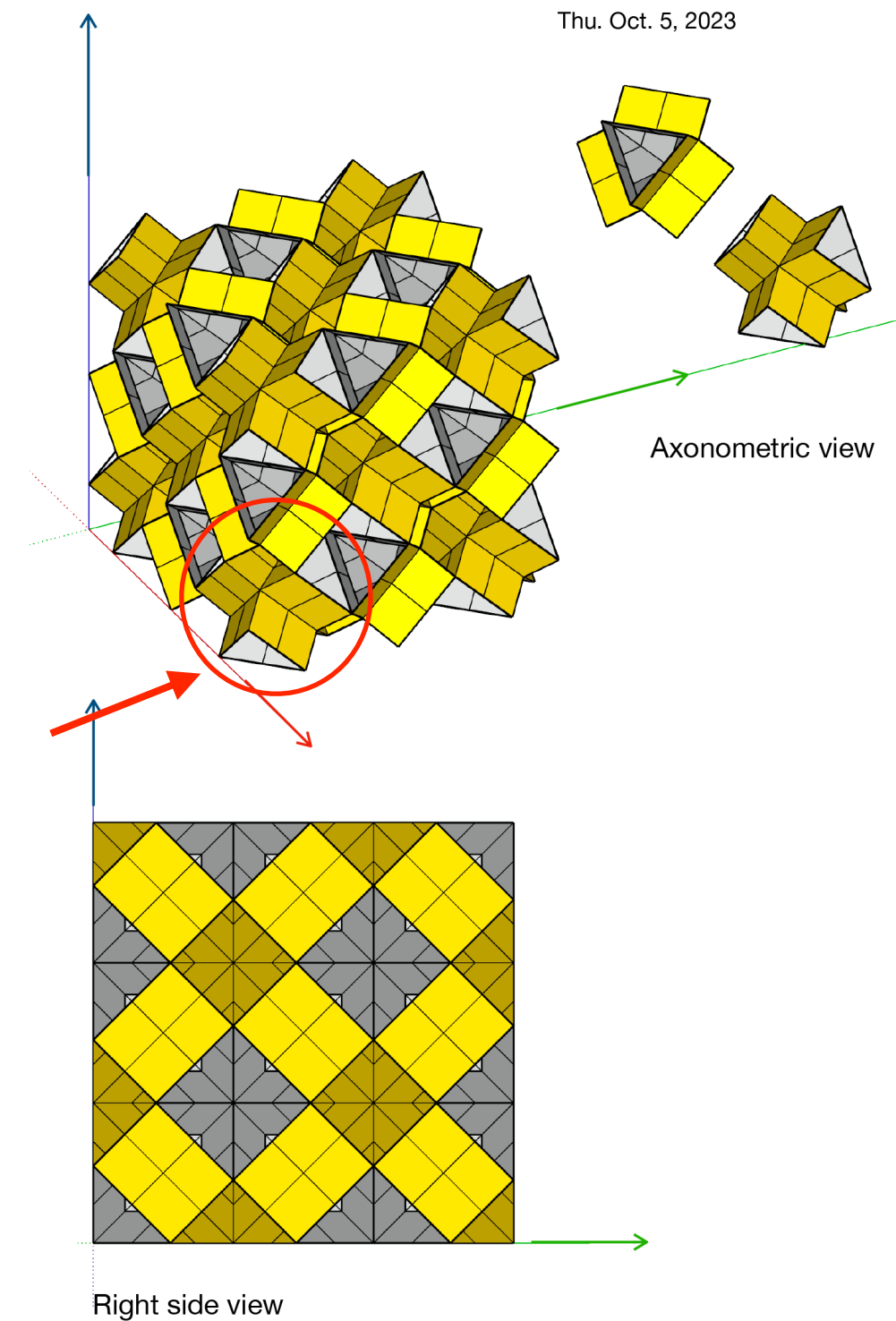
**STEP 2: HALVE THE EXTRUSION LENGTH**



### STEP 3: “PUSH-FIT” THE EXTRUDED PRISMS



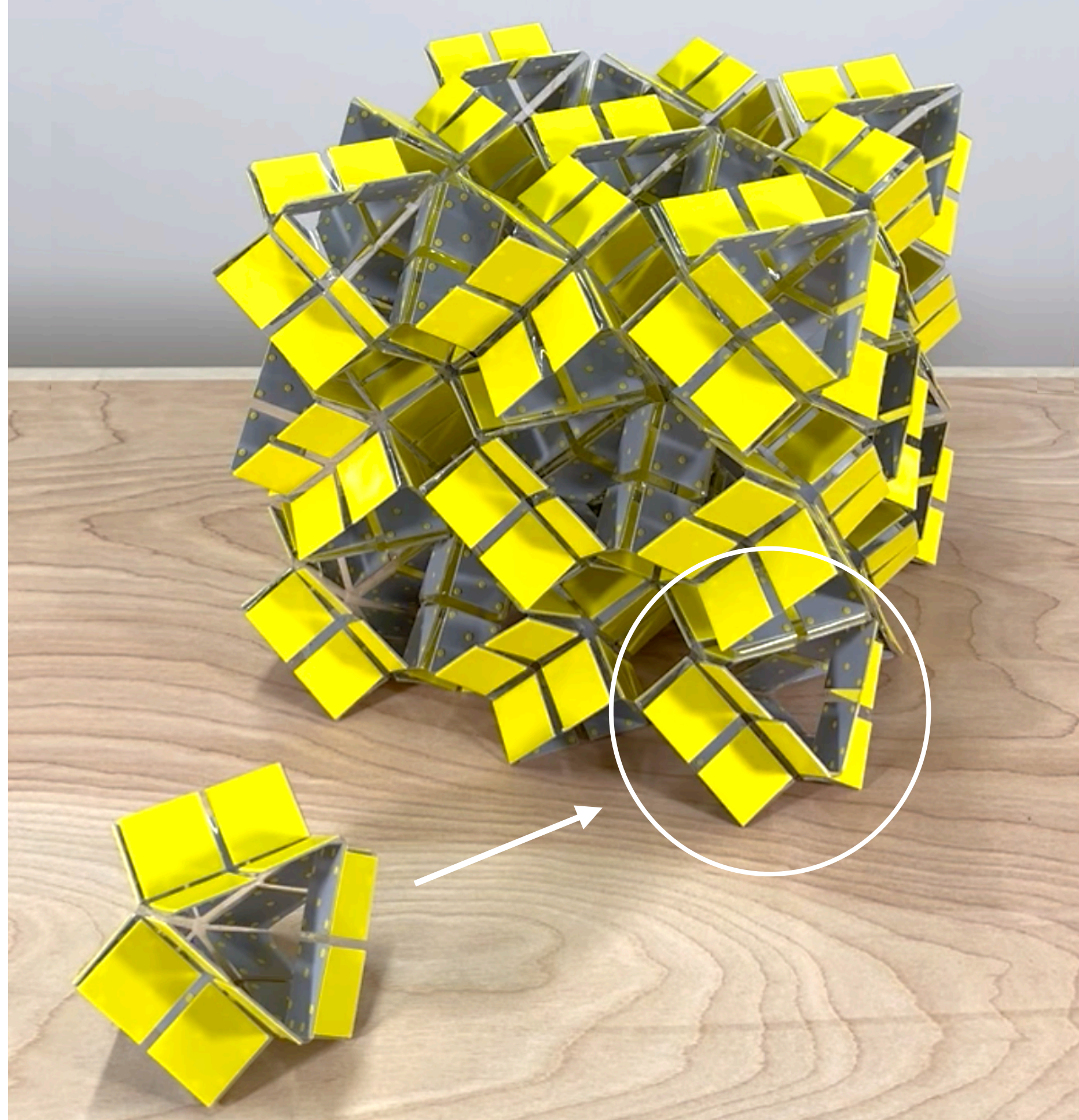
**MATERIAL #1**  
**TETRAHEDRA & OCTAHEDRA**  
**(FLAT-FOLDABLE)**





## MATERIAL #1 PHYSICAL MODEL

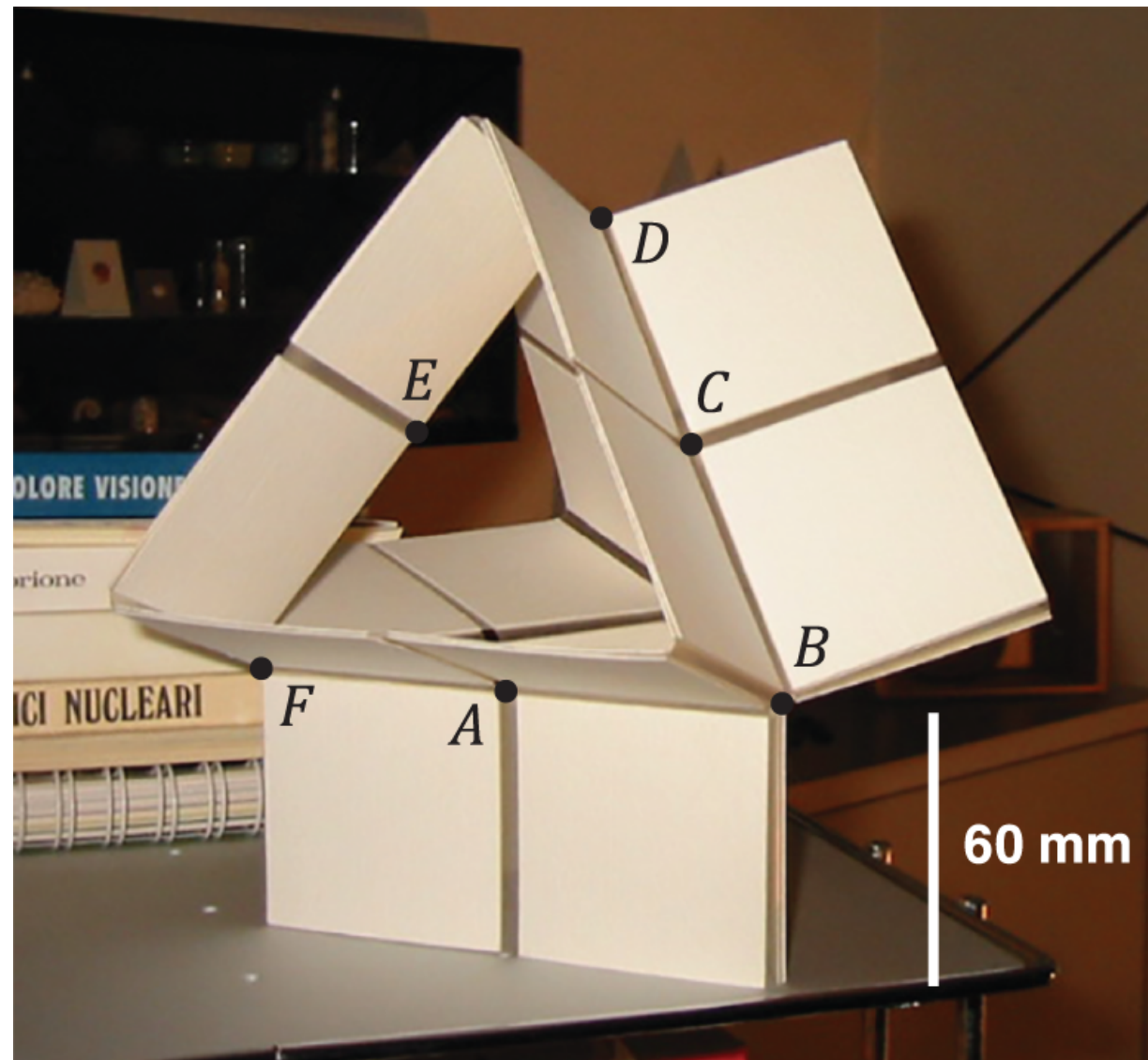
(Model fabrication:  
E. Montano, J. Minnick,  
D. Munechica, J. Cuellar,  
P. Trogu, 2023)





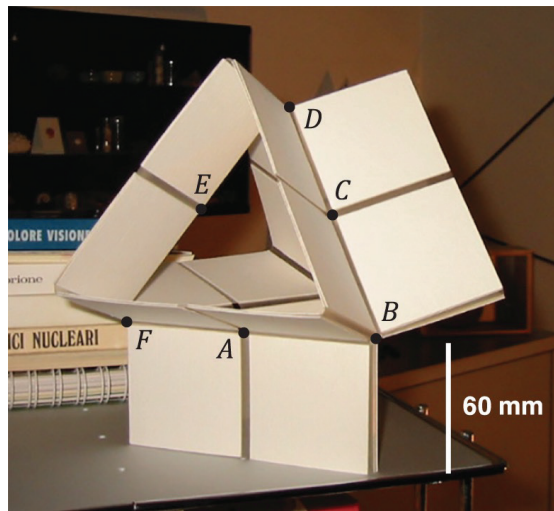
# EXTRUDED TETRAHEDRON:

FROM  
NON RIGIDLY FLAT-FOLDABLE  
TO RIGIDLY FLAT-FOLDABLE.



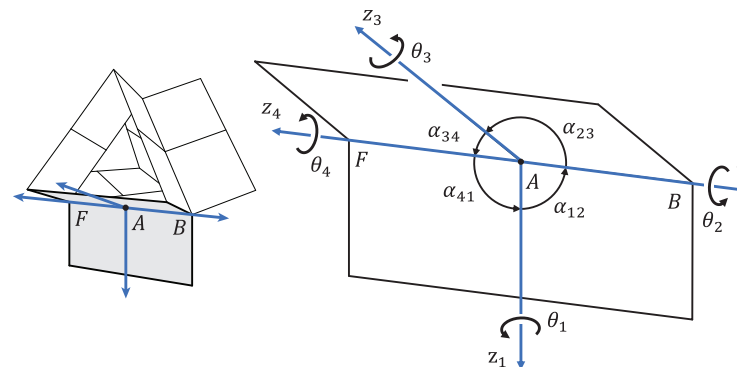
Original 1996 prismatic tetrahedron model with six representative vertices.

# Scarpa's transformable model (1996)

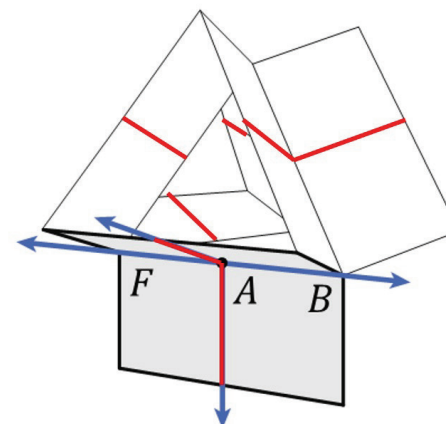


It is flat-foldable

## Rigid foldability analysis



D-H notation of the four-crease vertex  $A$ .



inactive creases

It is not rigidly foldable

Conditions:

$$\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = \frac{\pi}{2}$$

Closure equation (D-H):

$$Q_{21}Q_{32} = Q_{41}Q_{34}$$

where

$$Q_{(i+1)i} = \begin{bmatrix} \cos \theta_i & -\cos \alpha_{i(i+1)} \sin \theta_i & \sin \alpha_{i(i+1)} \sin \theta_i \\ \sin \theta_i & \cos \alpha_{i(i+1)} \cos \theta_i & -\sin \alpha_{i(i+1)} \cos \theta_i \\ 0 & \sin \alpha_{i(i+1)} & \cos \alpha_{i(i+1)} \end{bmatrix}$$

$$Q_{i(i+1)} = \begin{bmatrix} \cos \theta_i & \sin \theta_i & 0 \\ -\cos \alpha_{i(i+1)} \sin \theta_i & \cos \alpha_{i(i+1)} \cos \theta_i & \sin \alpha_{i(i+1)} \\ \sin \alpha_{i(i+1)} \sin \theta_i & -\sin \alpha_{i(i+1)} \cos \theta_i & \cos \alpha_{i(i+1)} \end{bmatrix}$$

Solution:

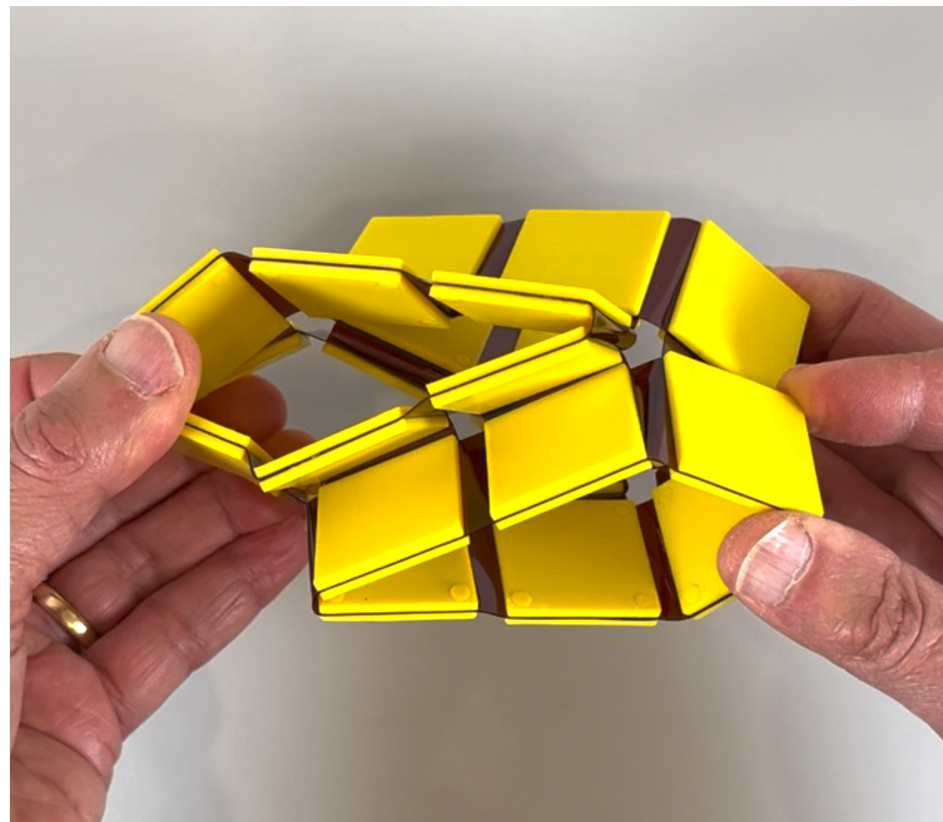
$$\theta_1 = \theta_3 = 0, \theta_2 = \theta_4; \\ \text{or } \theta_2 = \theta_4 = 0, \theta_1 = \theta_3.$$



## SCARPA'S ORIGINAL SHAPE: NON-RIGIDLY FOLDABLE



a



b

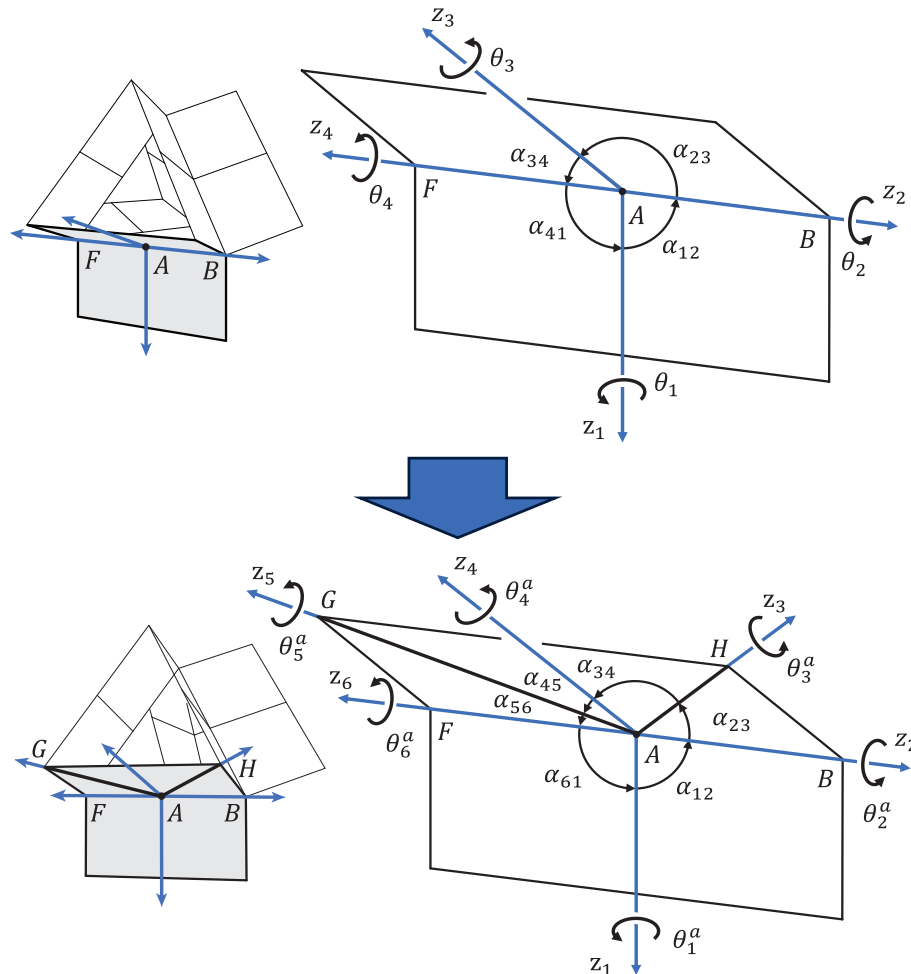


c

Non-rigid folding of prismatic tetrahedron: (a) Stable configuration; (b) Partially deformed configuration; (c) Flat-folded configuration.

(Model fabrication: H. Feng, W. Shi, P. Trogu, E. Montano, J. Minnick, G. Montalvo, T. Casanova, 2023)

### Step 1: add two creases at vertex $A$



Altered vertex  $A$  with two additional symmetric creases.

### Kinematics of the altered six-crease vertex

Conditions:

$$\alpha_{12} = \alpha_{61} = \frac{\pi}{2},$$

$$\alpha_{23} = \alpha_{34} = \alpha_{45} = \alpha_{56} = \frac{\pi}{4},$$

$$\theta_2^a = \theta_6^a, \theta_3^a = \theta_5^a.$$

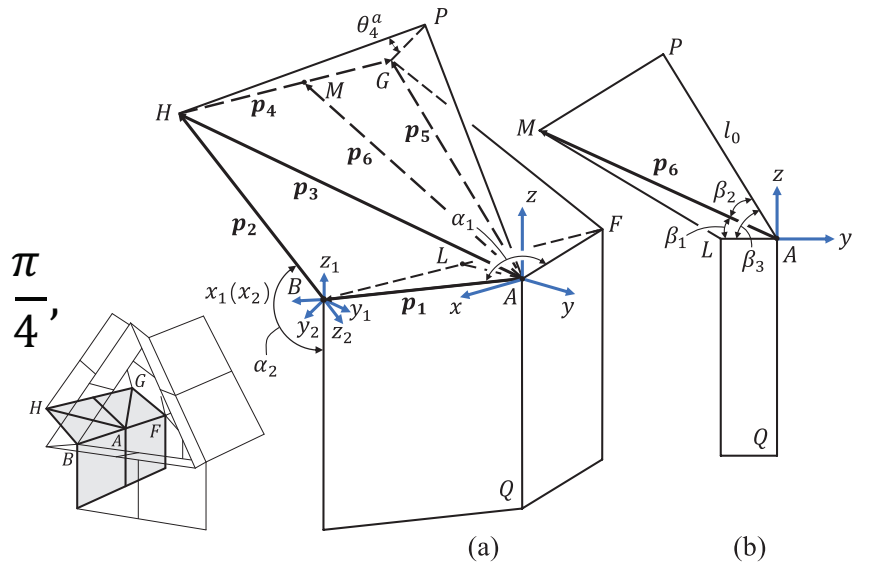
Assuming:

$$\theta_1^a = \alpha_1, \theta_2^a = \alpha_2.$$

Solution:

$$\theta_4^a = \arccos \left( 1 - 2 \left( \sin \left( \frac{\alpha_1}{2} \right) - \cos \left( \frac{\alpha_1}{2} \right) \sin(\alpha_2) \right)^2 \right);$$

$$\theta_5^a = f(\alpha_1, \alpha_2).$$

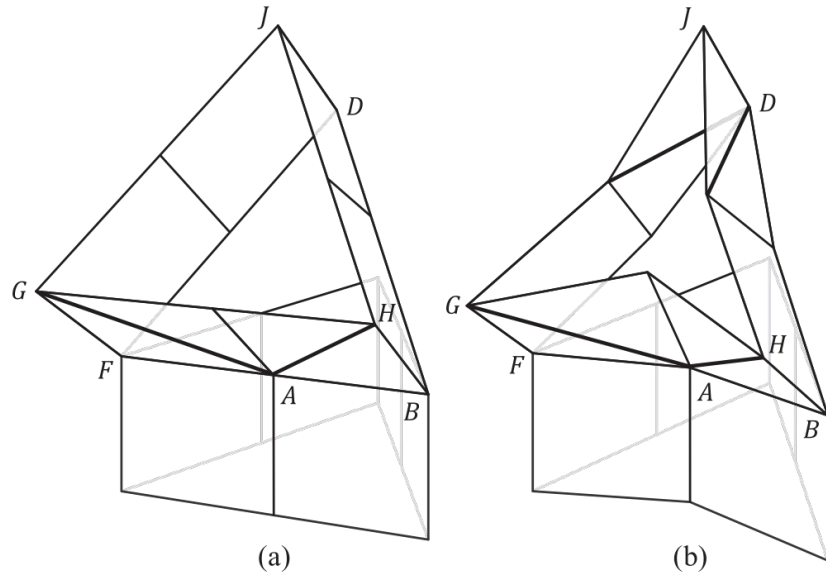


(a) Coordinate system setup; (b) Side view from the y-A-z plane.

The altered six-crease vertex  $A$  has 2 Dof



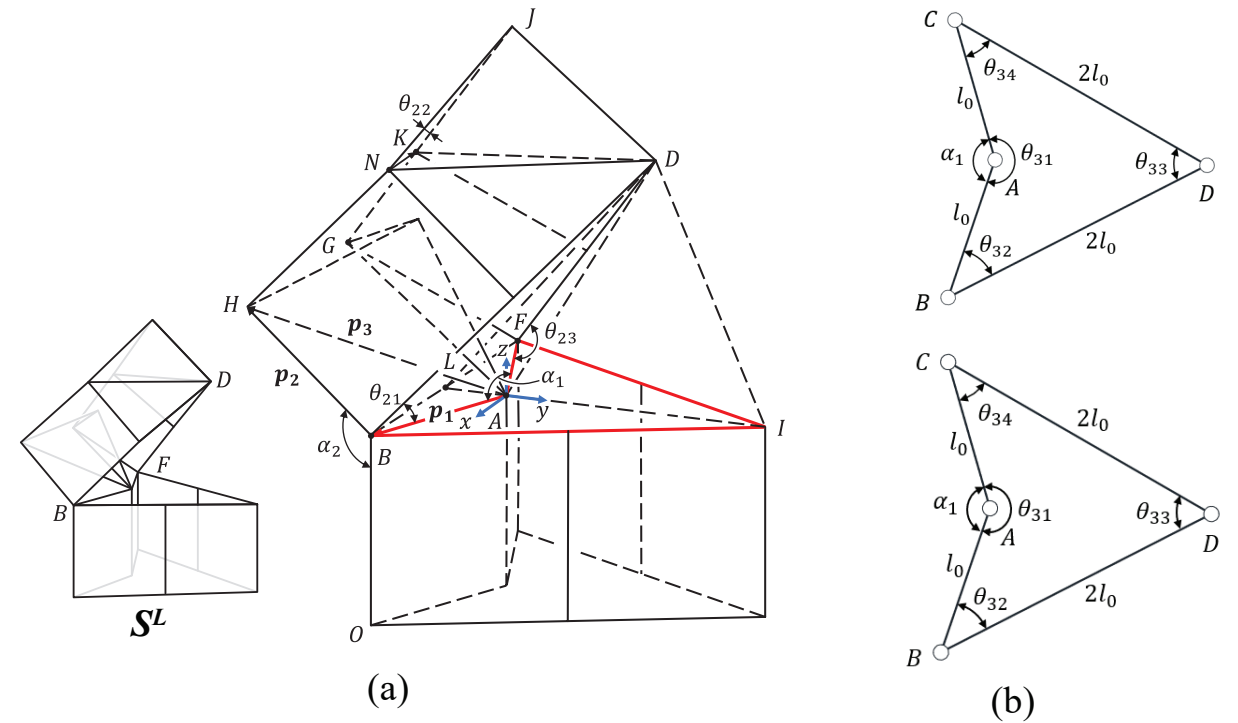
## Step 2: add two creases at vertex $D$



Double triangular prism group with (a) only one altered vertex  $A$ ; (b) altered central vertex  $A$  and corner vertex  $D$ .

The creases  $AG$  and  $AH$  added in step 1 are inactive in (a).

## Kinematics of double triangular prism group

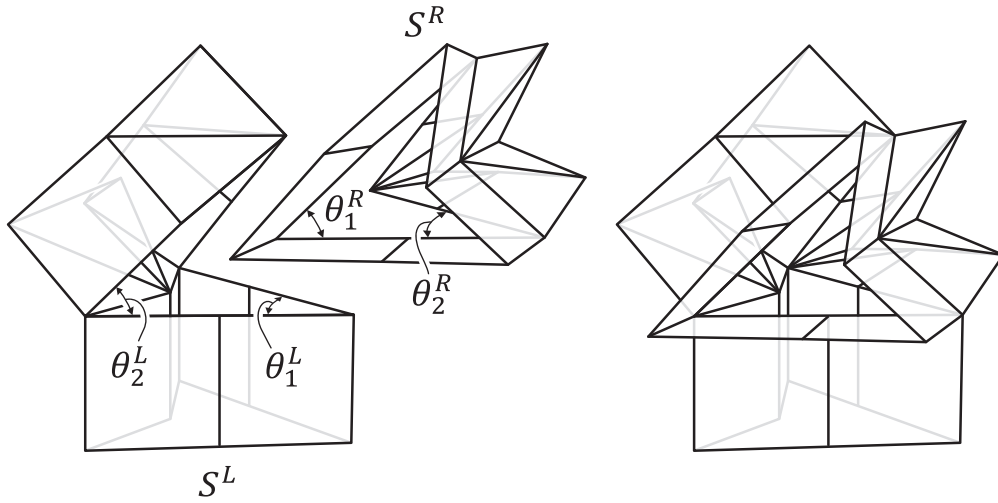


(a) Schematic diagram of two triangular prisms; (b) projection of the lower triangular prism along the line  $BO$ .

$$\alpha_1, \alpha_2 \rightarrow \begin{cases} \theta_3^a, \theta_4^a, \theta_5^a, \theta_6^a, \\ \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \\ \theta_{21}, \theta_{22}, \theta_{23}. \end{cases}$$

Double triangular prism group has 2 Dof

### Step 3: compatibility analysis

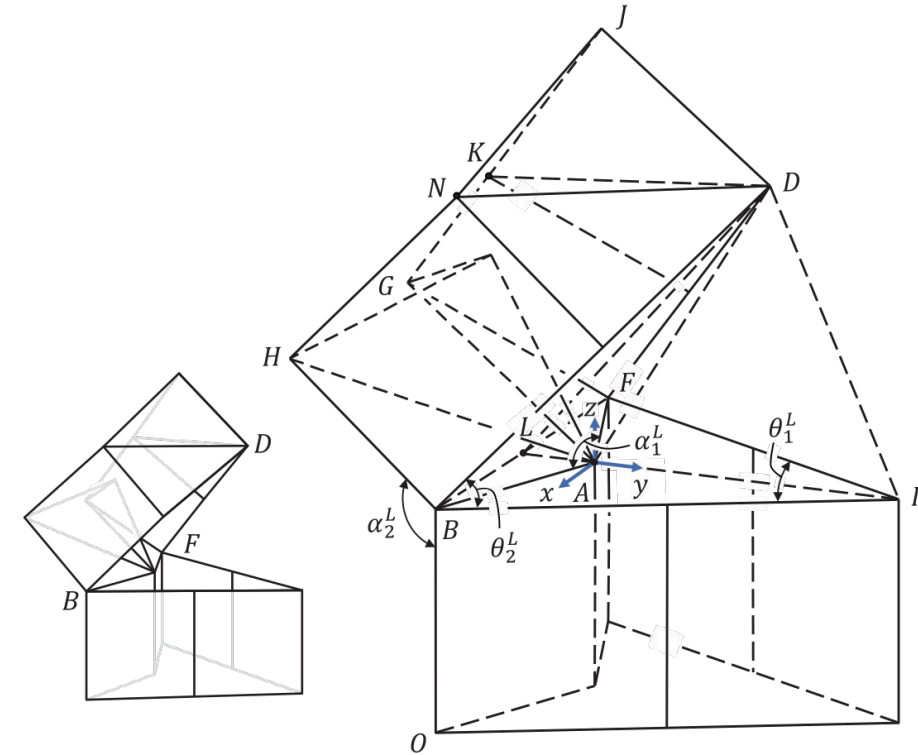


The two groups in exploded and assembled views

The compatible conditions of the whole structure are that the size lengths of the two groups are equal and

$$\theta_1^L = \theta_2^R, \theta_2^L = \theta_1^R.$$

### Kinematics of the assembly angles



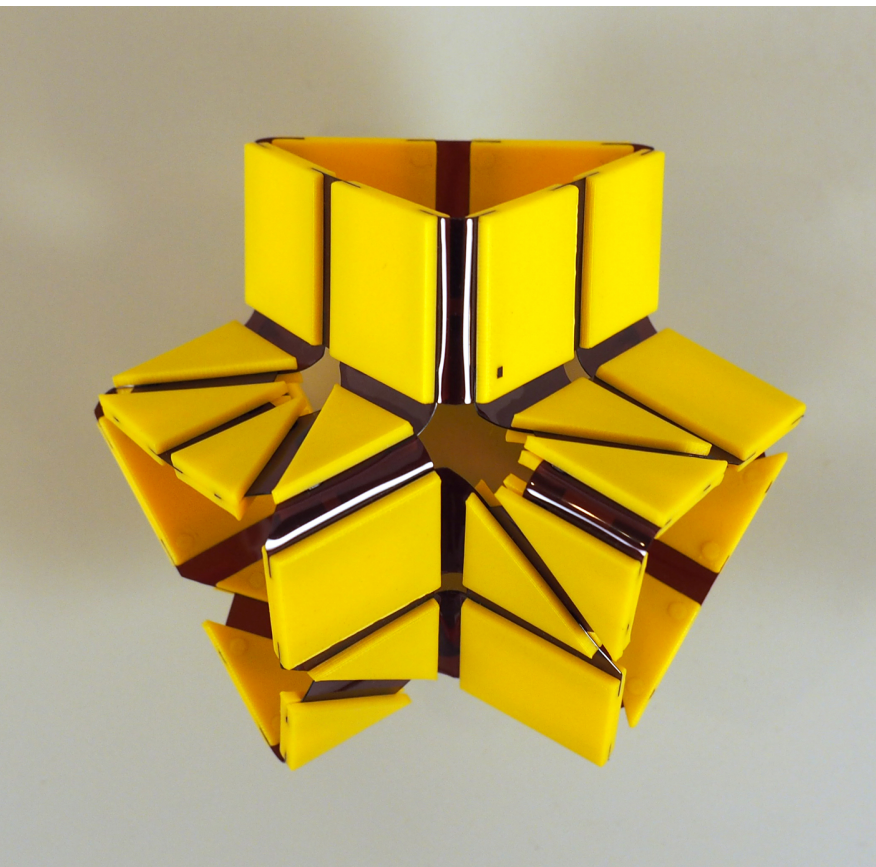
Relationship between the configuration angles and assembly angles.

Solution:

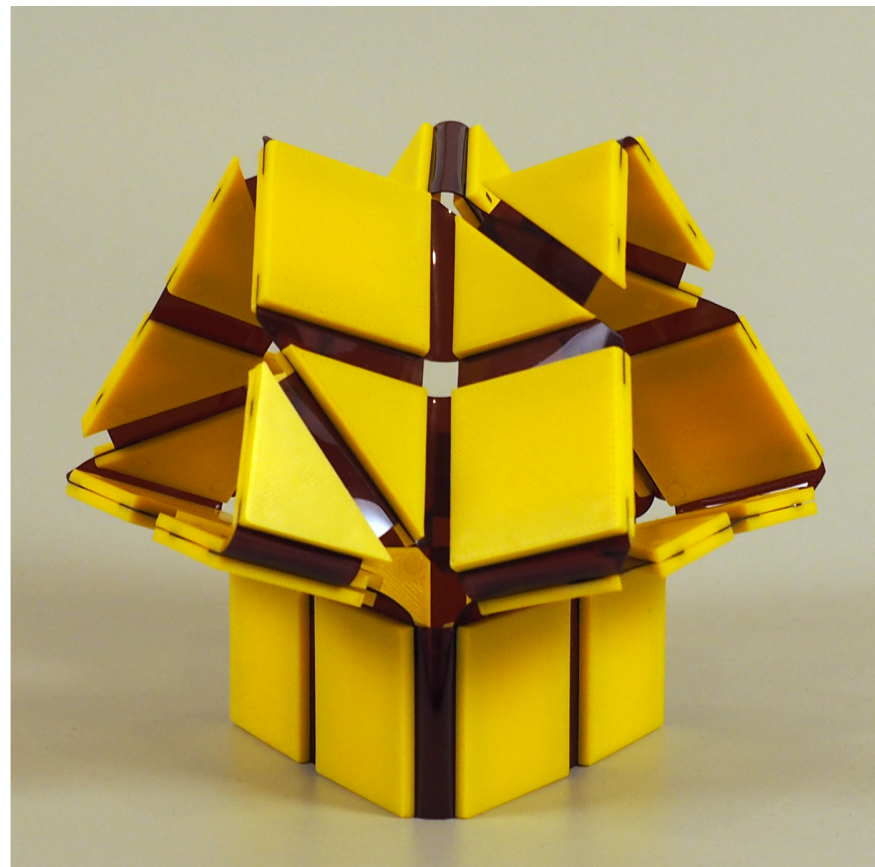
$$\theta_1^L = f(\alpha_1),$$

$$\theta_2^L = f(\alpha_1, \alpha_2).$$

## MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES



Top view



Front view



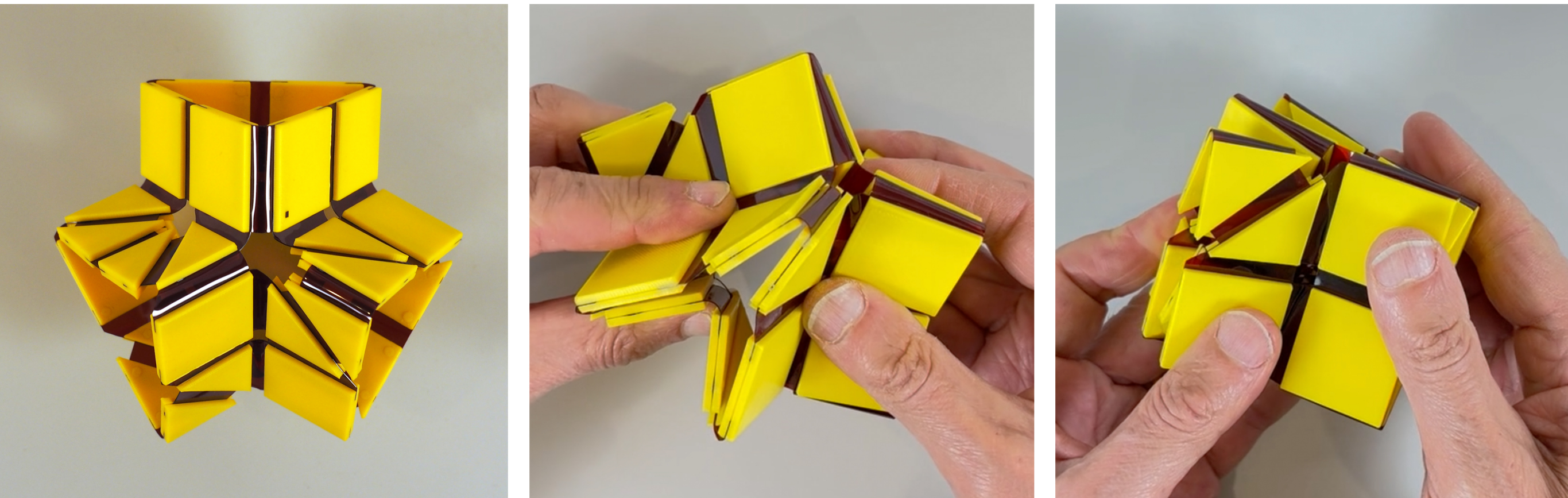
Right-side view

Orthographic views of the altered auxetic metamaterial.

(Model fabrication: H. Feng, W. Shi, P. Trogu, E. Montano, J. Minnick, G. Montalvo, T. Casanova, 2023)



## MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES



Rigid folding process of the altered auxetic metamaterial.

## Giorgio Scarpa

Italian designer, bionics and topology researcher, teacher, and artist.

Profile and videos by **Pino Trogu**, San Francisco State University [trogu at sfsu dot edu]

The short videos below refer to the topics of Scarpa's two books. The first is a bionic study of the mouth apparatus of the sea urchin, also known as Aristotle's Lantern, after the first detailed study of it by the Greek philosopher. Recently new prototypes of a biopsy harvester and a ground sampler were inspired by Scarpa's model of the urchin. This new instrumentation is described in an article published in 2016 by the journal Leonardo (MIT). Click links below to download those articles. The PDFs of the bionics books are at right. The English is an unpublished draft translation; the Italian is the original text and images.

The second video shows one of the many modular chains described in the rotational geometry book, which focuses on rotational movement as a basic form generating process. Scarpa dissects the five Platonic solids and other solids into chains of hinged triangular pyramids that fold back into their enclosure cells. The PDFs of the geometry books are at right. The English is an unpublished draft translation; the Italian is the original text and images.

Both bionics and geometry books were published as part of a now out-of-print series called Design Notebooks, edited by the late Italian designer Bruno Munari. The covers of the books in that series are shown below. The other videos show more topological and bionic studies by Scarpa, including DNA models and studies of muscle cells.

This page was last updated on Sunday Feb. 6, 2022.

On July 29, 2021, the website was moved from <http://online.sfsu.edu/trogu/scarpa/> to <https://res.trogu.com/scarpa>



Bionic Model of Aristotle's Lantern  
Video length: 1:12 | Video: Pino Trogu, 1994.

### Interviews

[Leonardo Book Club: Live discussion with Pino Trogu, author, "Giorgio Scarpa's Model of a Sea Urchin Inspires New Instrumentation"](#)  
Length: 58:30 | April 24, 2019.

### Articles

Pino Trogu  
[Giorgio Scarpa's Model of a Sea Urchin Inspires New Instrumentation](#),  
Leonardo Journal, 52.2, 2019.  
Free article download from MIT Press website:  
[doi:10.1162/LEON\\_a\\_01384](https://doi.org/10.1162/LEON_a_01384).



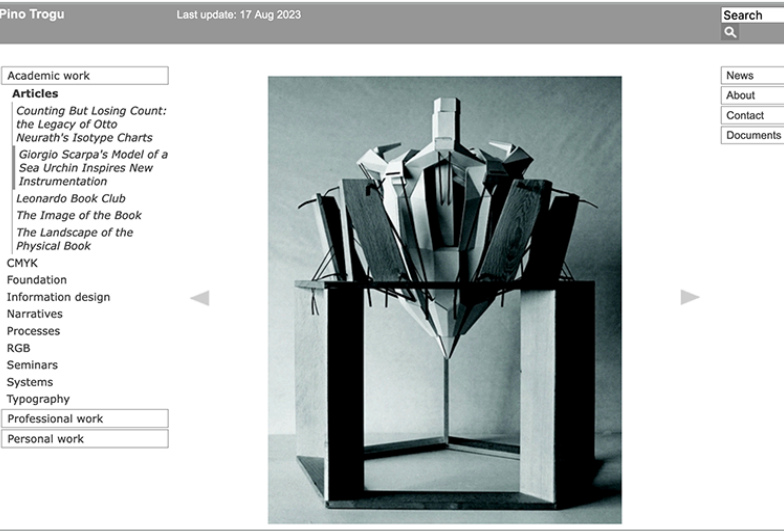
Download PDF of complete Bionic Models book.  
Unpublished English translation of Italian Edition: Modelli di Bionica, 1985.  
Translated by Pino Trogu. 120 pages. File size: 38MB.



Scarica PDF completo del libro Modelli di Bionica Zanichelli, Bologna, 1985. 120 pagine. File size: 50MB.

LINKS

[res.trogu.com/scarpa](https://res.trogu.com/scarpa)



[trogu.com](https://trogu.com)

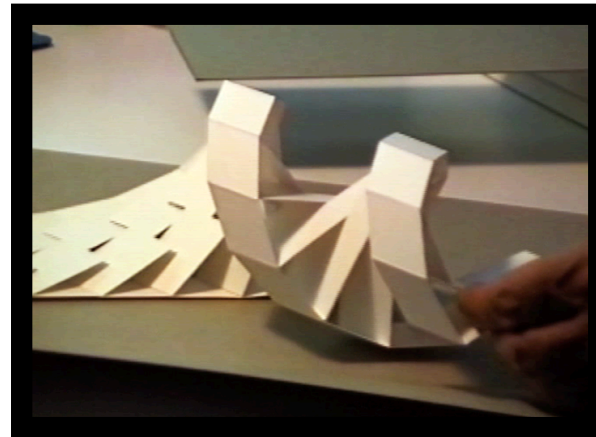


[lorenzobocca.com/giorgioscarpa](https://lorenzobocca.com/giorgioscarpa)

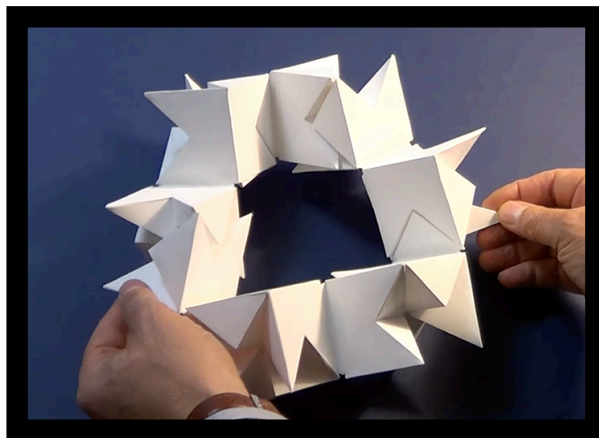




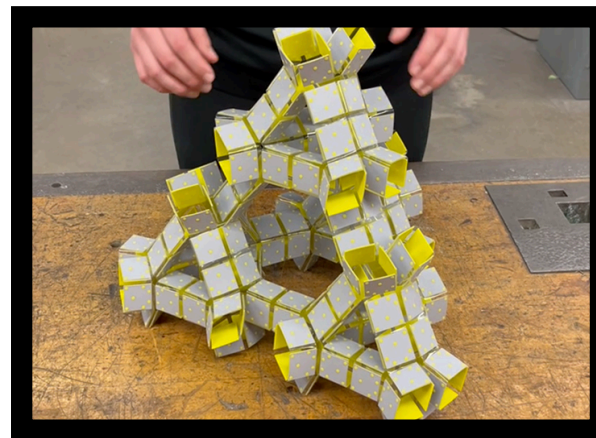
[Scarpa: Aristotle's Lantern](#)



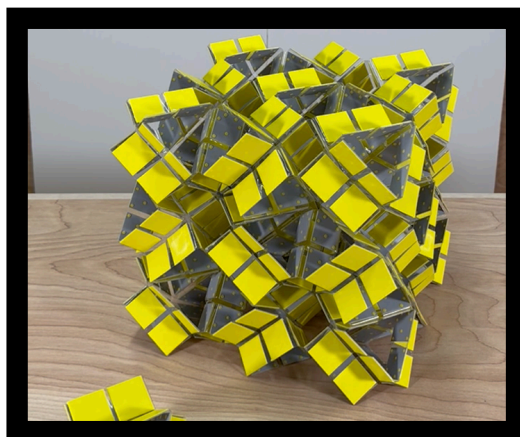
[Scarpa: DNA Model](#)



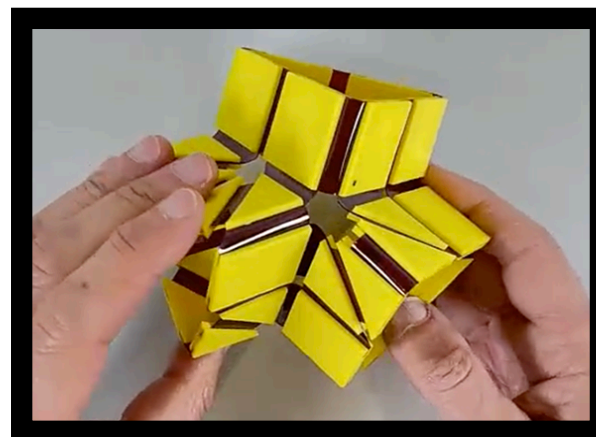
[Scarpa: Hexahedral Chain](#)



[Trogu: Material #6](#)



[Trogu: Material #1](#)

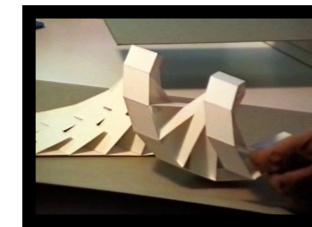


[Feng, Shi, Trogu, & Dai: Auxetic Metamaterial](#)

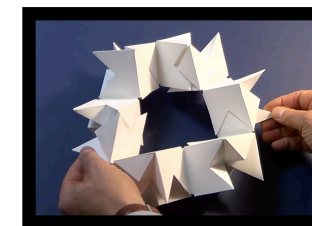
## VIDEOS (online/local)



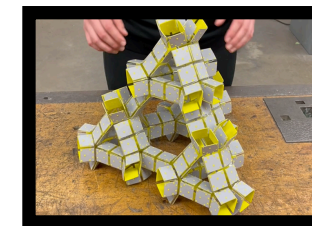
[Scarpa: Aristotle's Lantern](#)



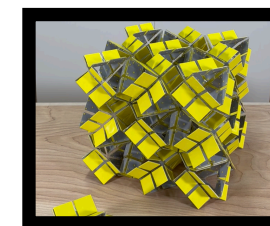
[Scarpa: DNA Model](#)



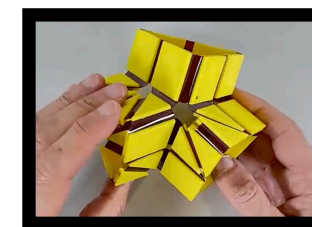
[Scarpa: Hexahedral Chain](#)



[Trogu: Material #6](#)



[Trogu: Material #1](#)



[Feng, Shi, Trogu, & Dai: Auxetic Metamaterial](#)

# THANK YOU!

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<sup>a</sup> Southern University of Science and Technology, Shenzhen, China  
<sup>b</sup> San Francisco State University, U.S.A.  
Corresponding author – email: trogu@sfsu.edu (P.Trogu)

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[Presentation Slides – PDF 32 MB](#)

CONTACT: [TROGU@SFSU.EDU](mailto:TROGU@SFSU.EDU)

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