Huijuan Feng[a], Wujie Shi[a], Pino Trogu[b], and Jian S. Dai[a], *Fellow, IEEE*

KINEMATIC MODELING OF A FLAT-FOLDABLE AUXETIC METAMATERIAL

THE 6TH INTERNATIONAL CONFERENCE ON RECONFIGURABLE MECHANISMS AND ROBOTS (REMAR 2024) – CHICAGO, U.S.A. JUNE 23–26, 2024

MONDAY, JUNE 24, 2024 – 2:30 PM

[a] Southern University of Science and Technology, Shenzhen, China
[b] San Francisco State University, U.S.A.
Corresponding author – email: trogu@sfsu.edu (P. Trogu)

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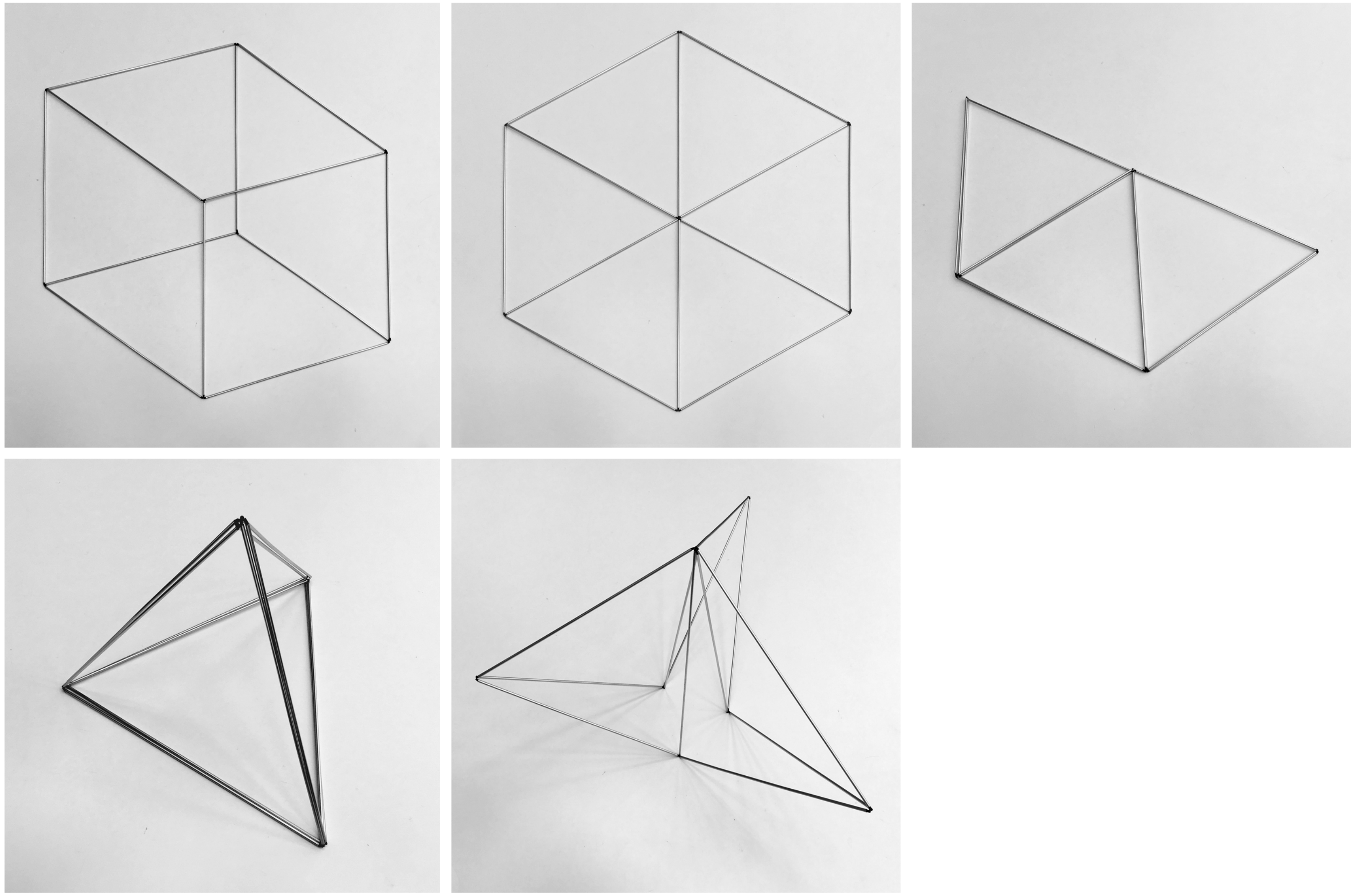
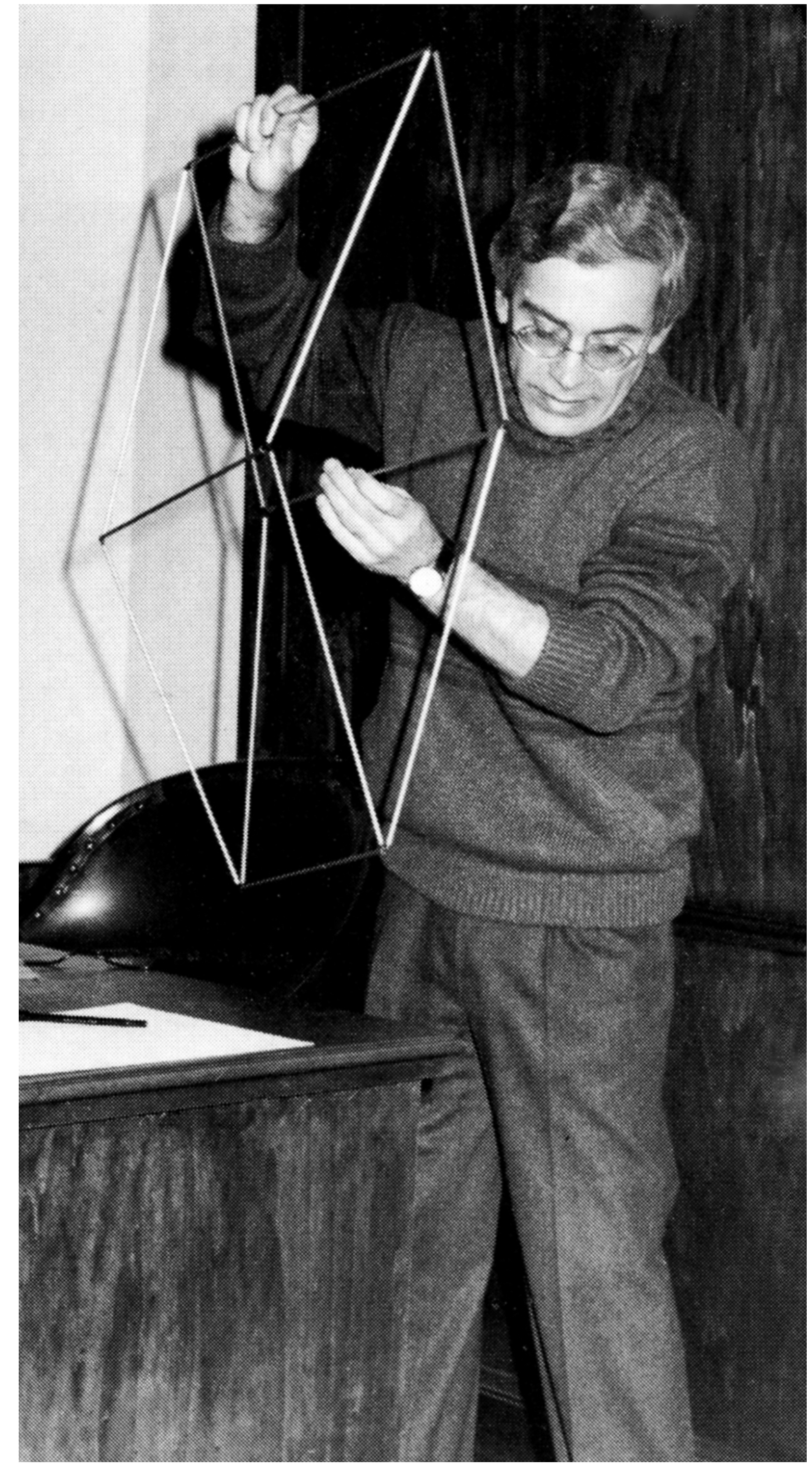
IEEE Robotics & Automation Society

IFToMM

GIORGIO SCARPA 1938–2012

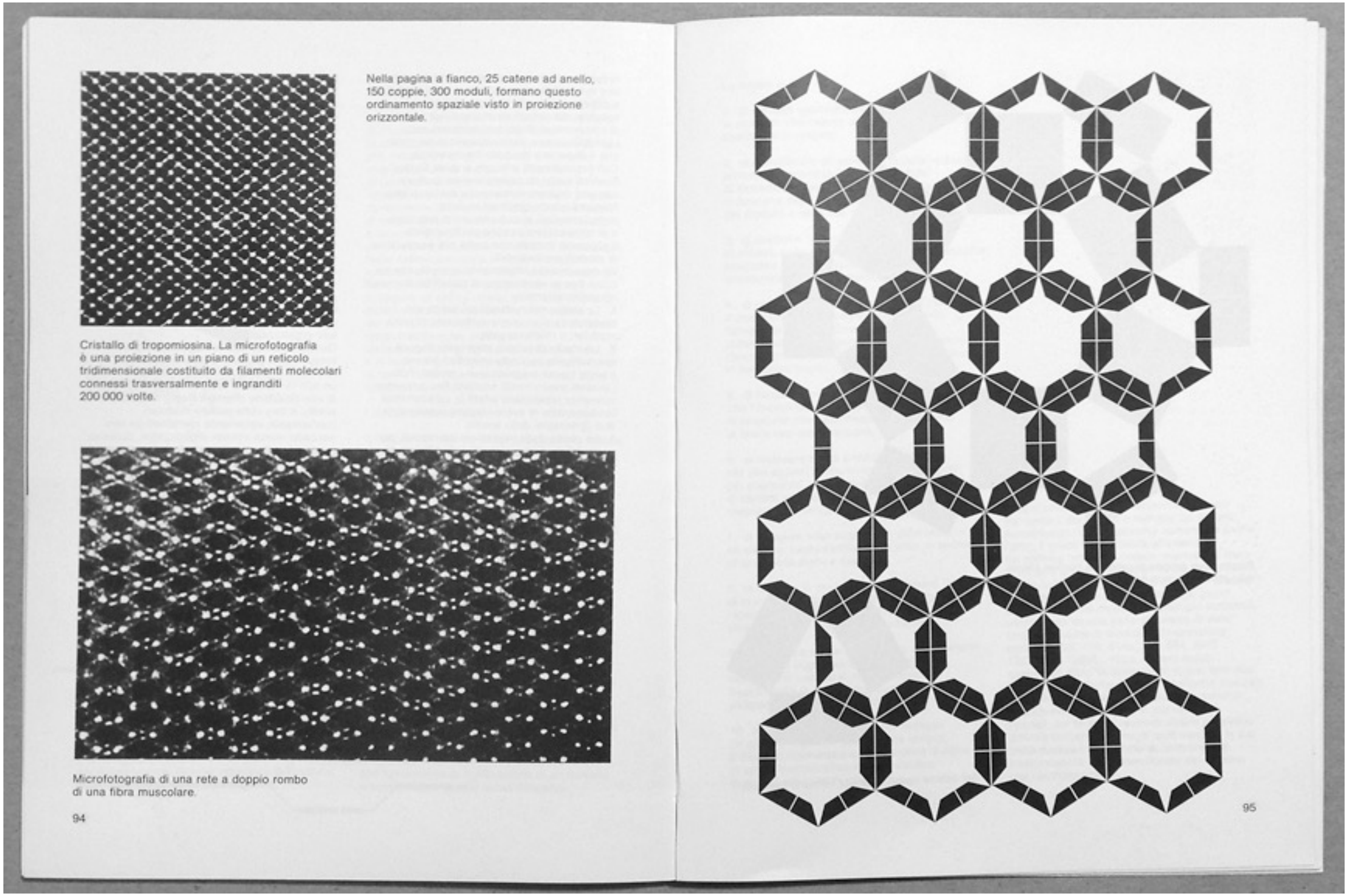
TRANFORMABLE CUBE, 1965





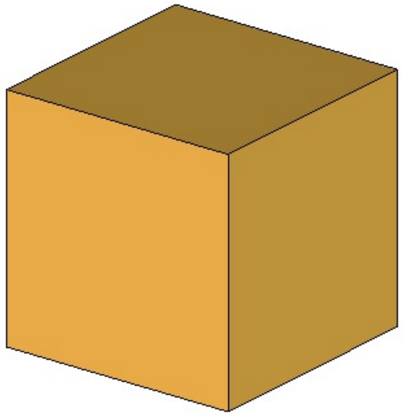
GIORGIO SCARPA

"TRANSFORMABLE FABRICS"

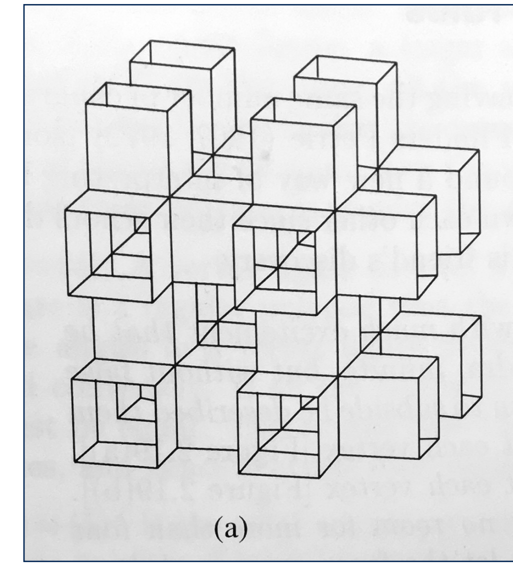


(G. Scarpa, Models of Rotational Geometry, 1978)

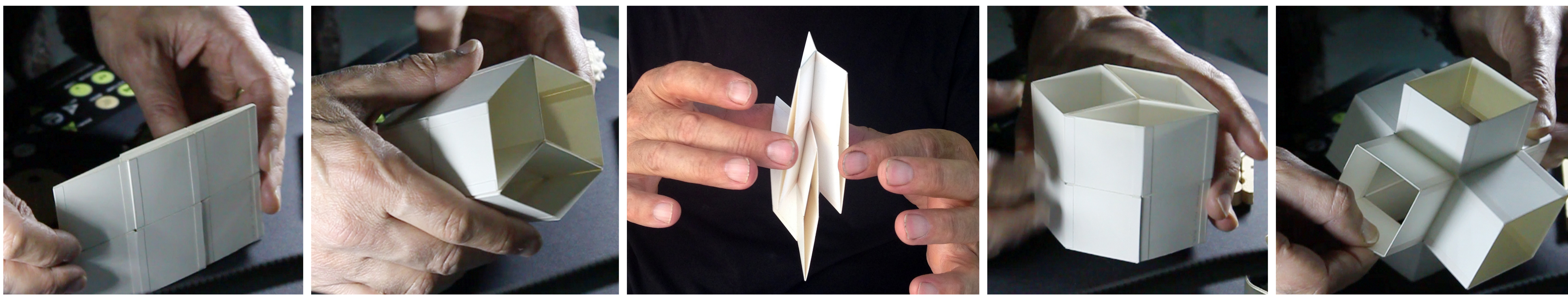
TWO
TRANSFORMABLE SHAPES:
“FIGURE TRASFORMABILI”
1996

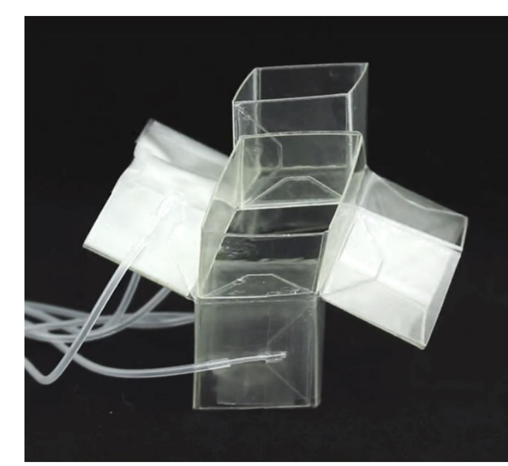


J. F. PETRIE
REGULAR HONEYCOMB, 1926



G. SCARPA, 1996



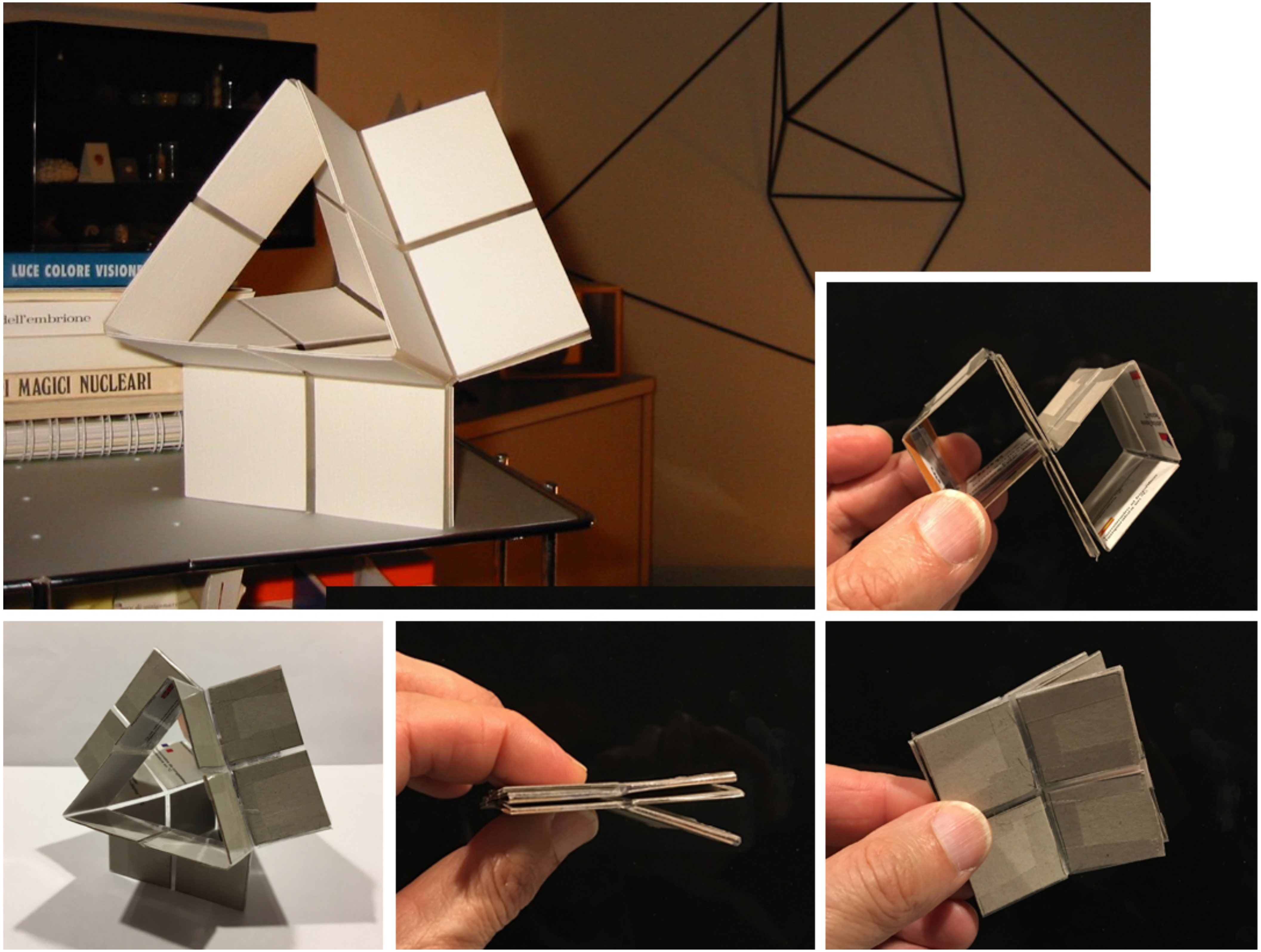


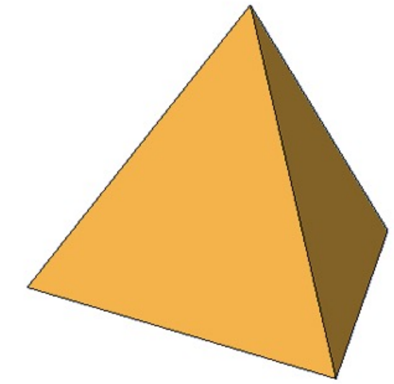
B. OVERVELDE *ET AL*, 2016

"I hope you like the transformable shape you will find in the package. But it was something else that I wanted to send you, a paper object that I built in three different versions but which still has more than one problem to be resolved."

Letter from Giorgio Scarpa to Pino Trogu
July 28, 1996

G. SCARPA – SPLIT EXTRUSION TETRAHEDRON – 1996





(REPLICA)
P. TROGU
2017

METAMATERIALS:

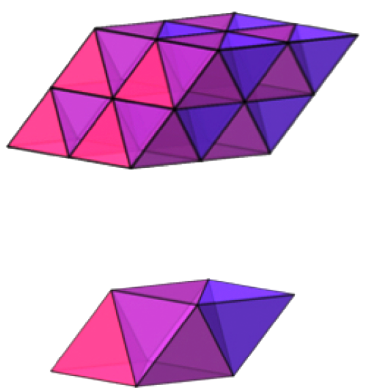
RIGID,

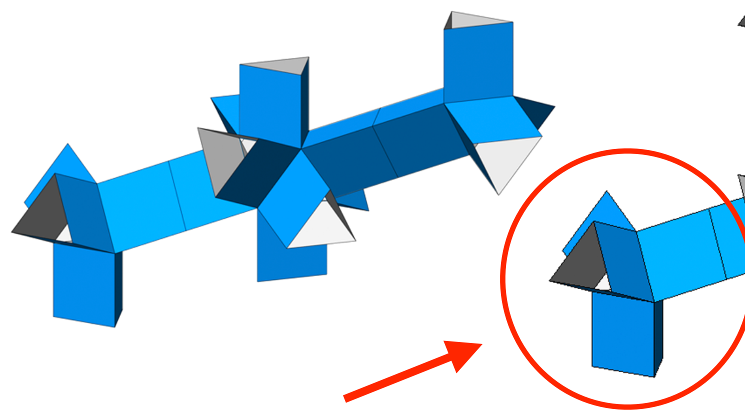
NON-RIGIDLY FOLDABLE,

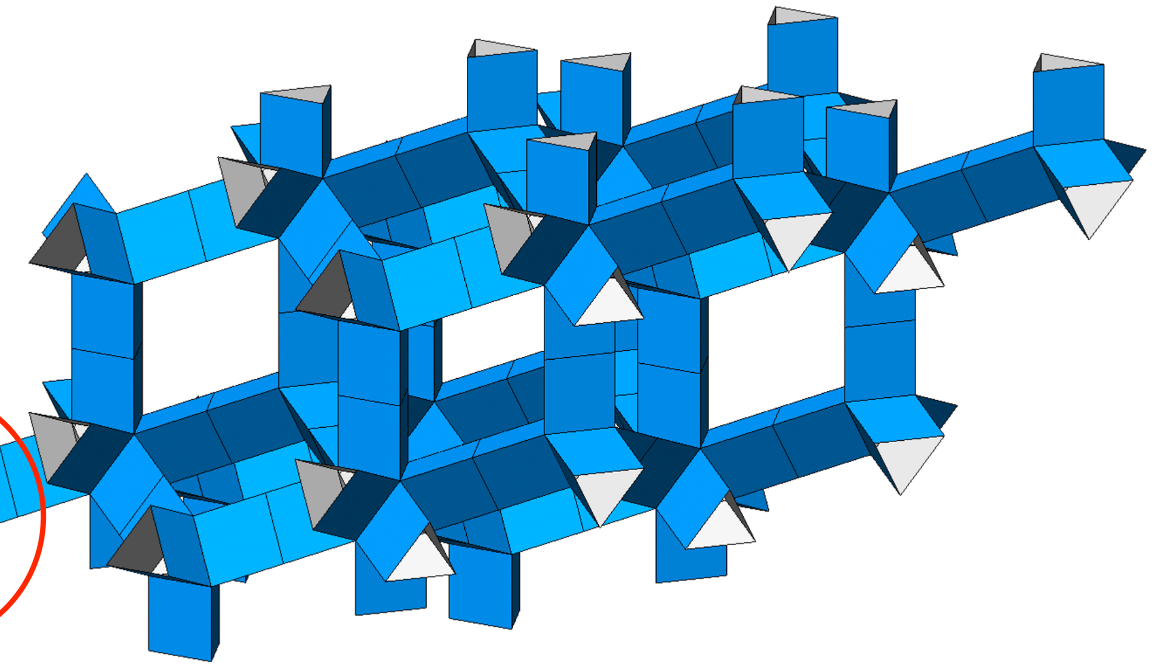
& RIGIDLY FOLDABLE.

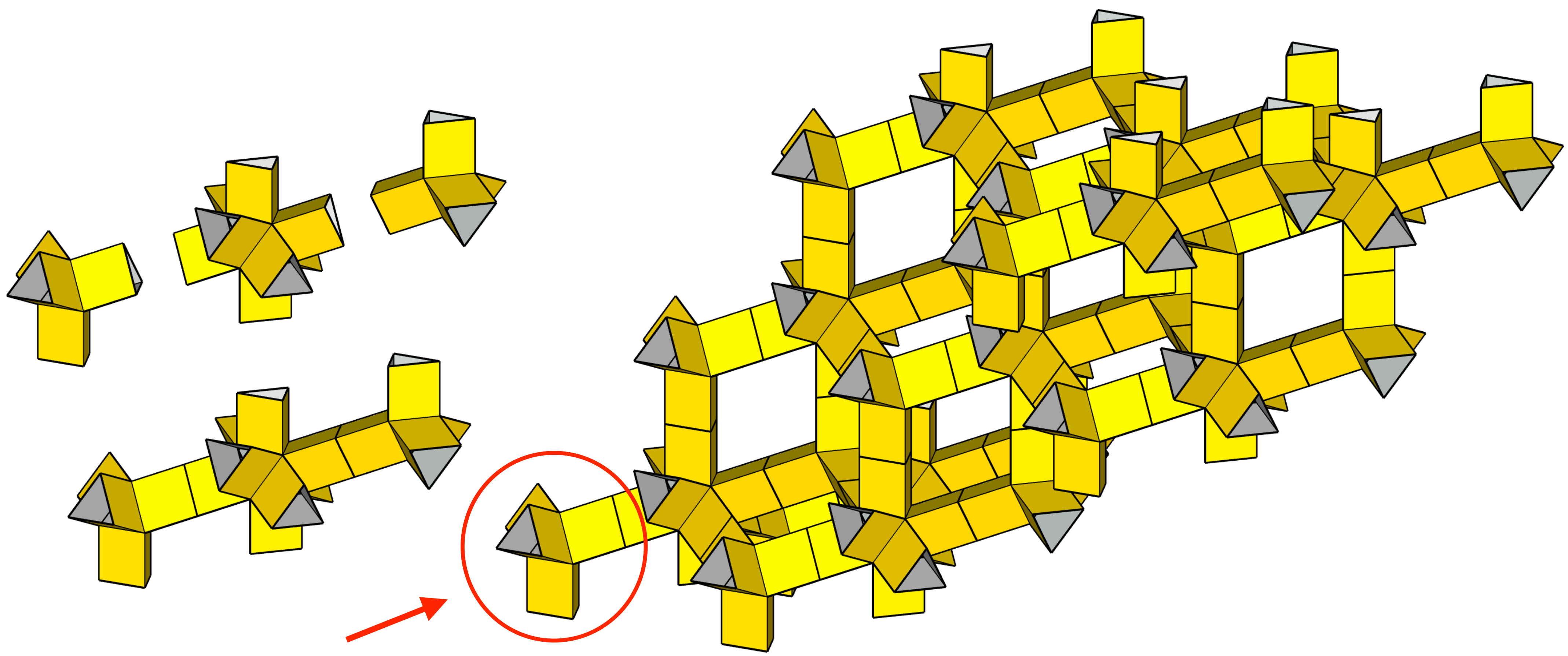
MATERIAL #1

TETRAHEDRA & OCTAHEDRA (RIGID)



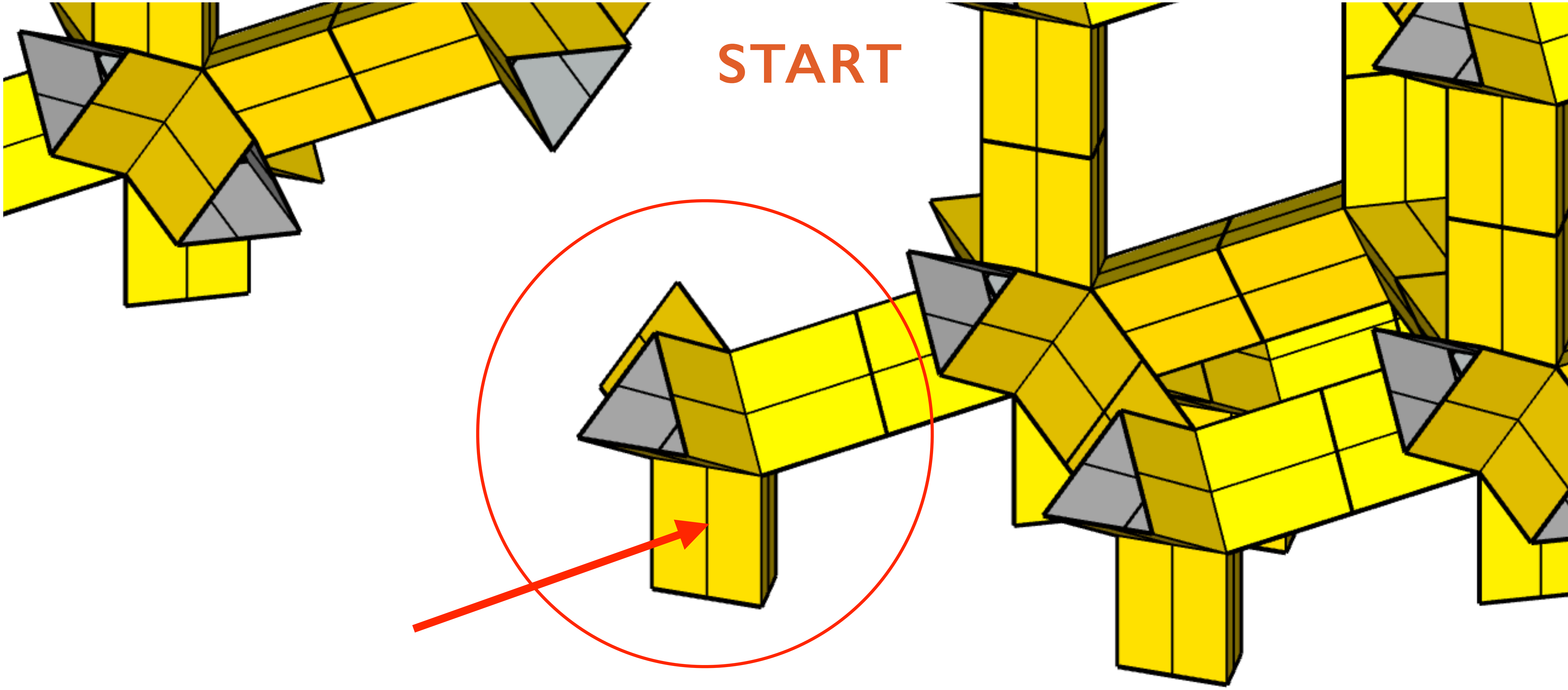




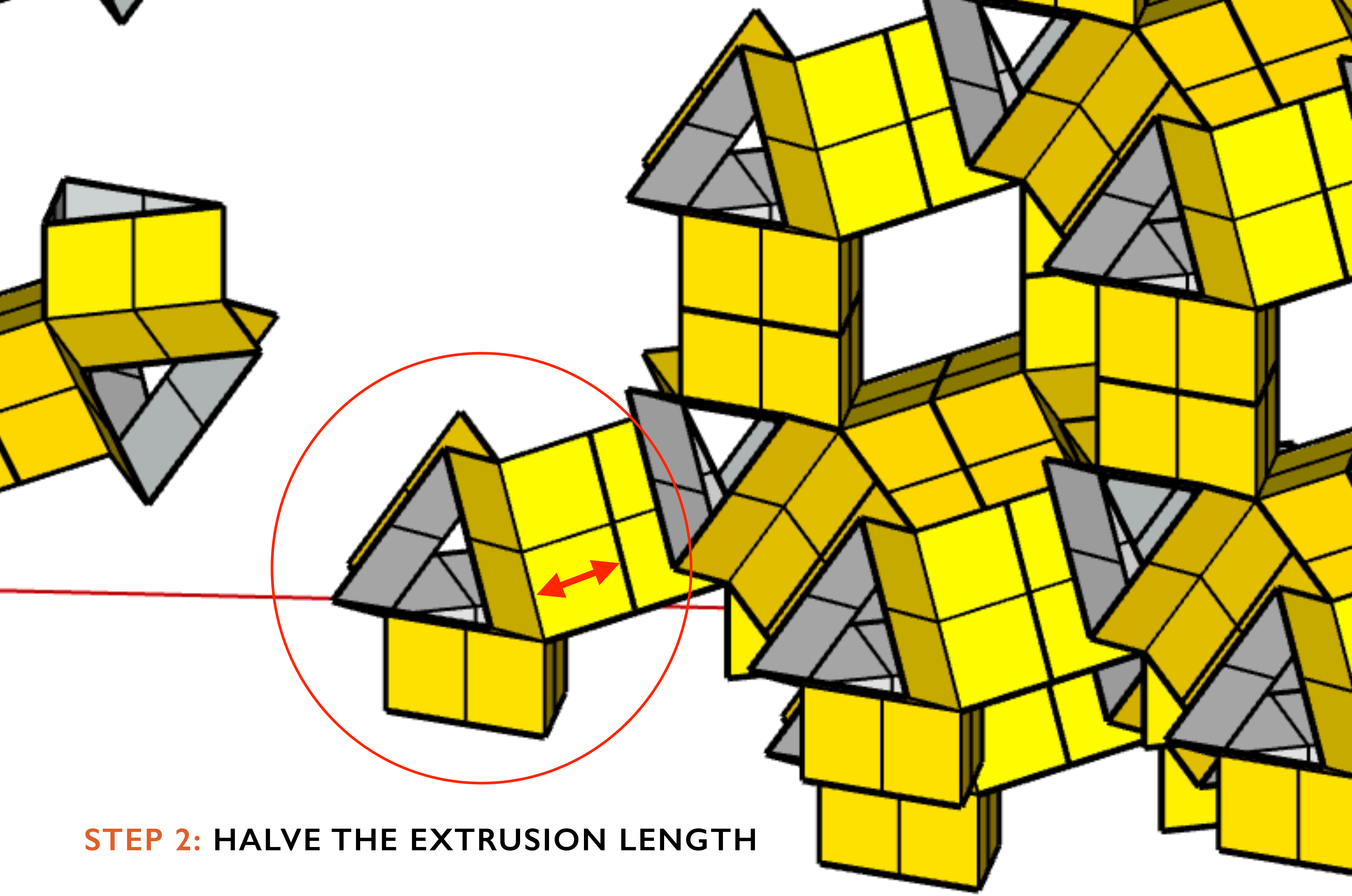


MATERIAL #1

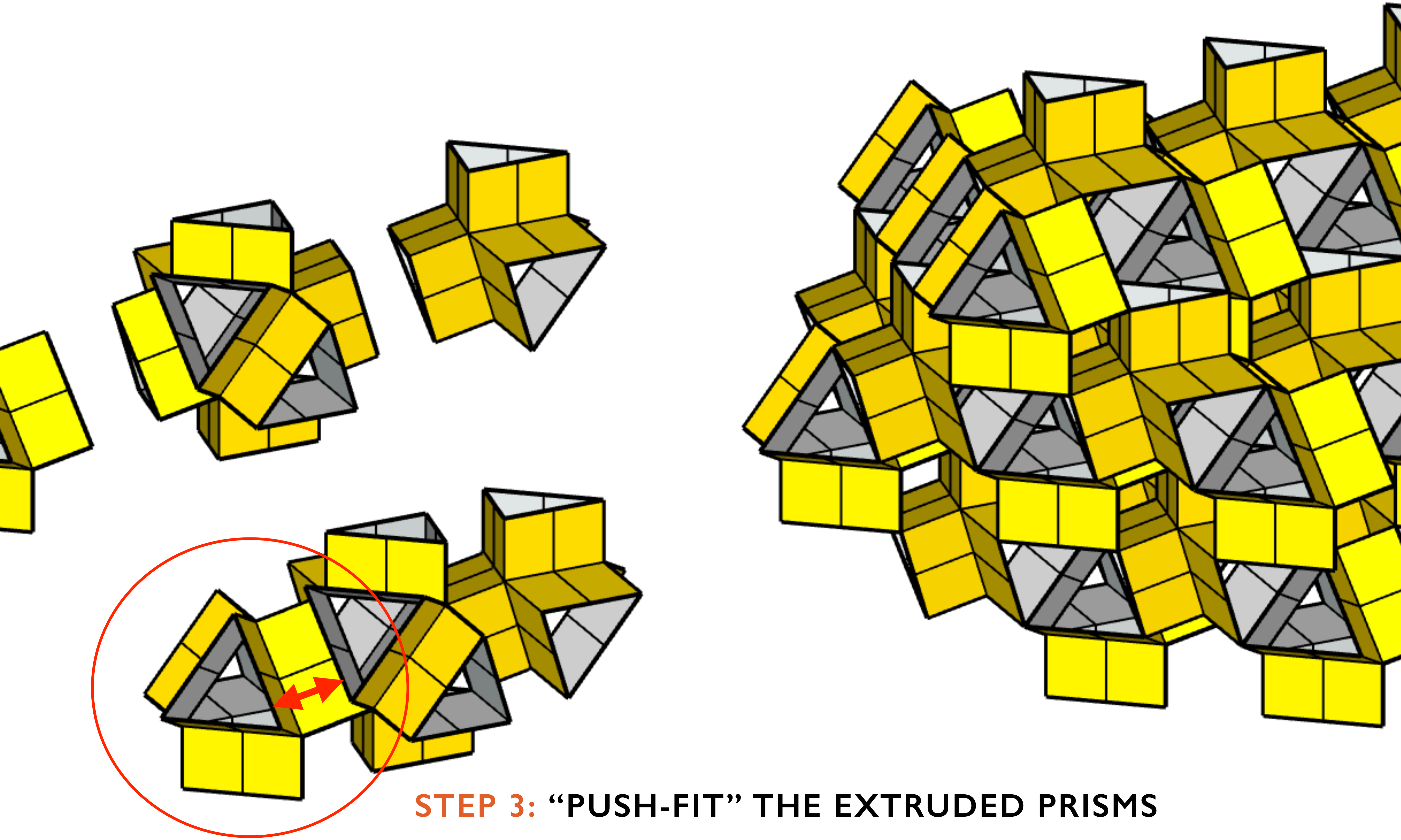
TETRAHEDRA & OCTAHEDRA (FLAT-FOLDABLE)



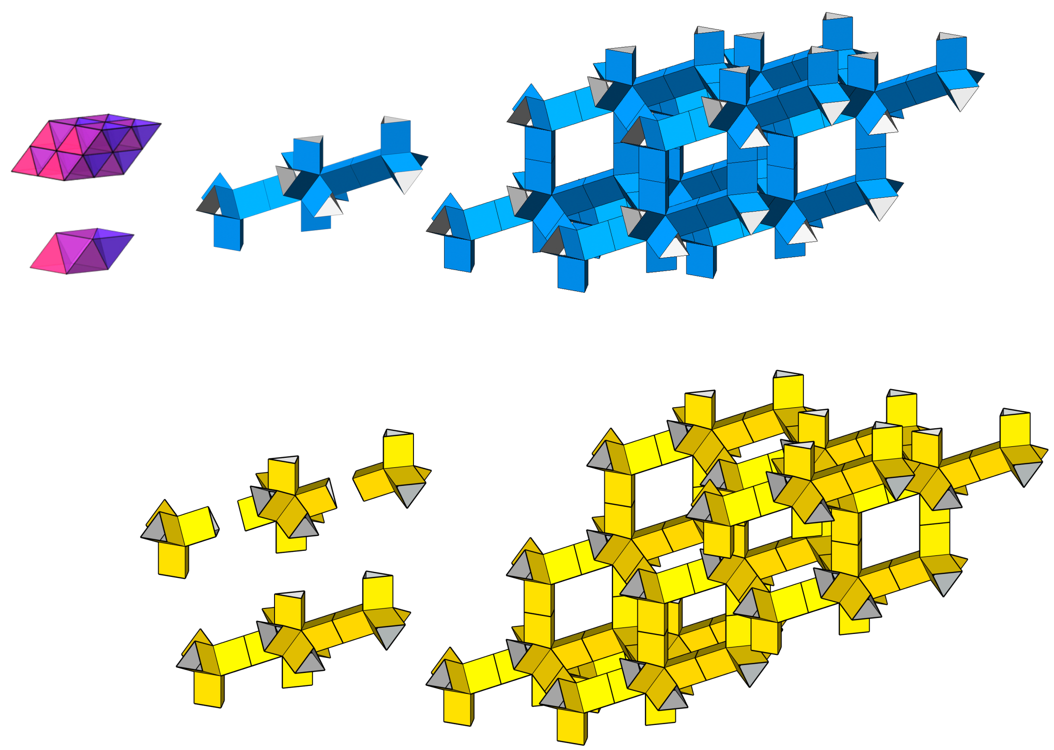
STEP 1: BISECT EXTRUDED PRISM WALLS

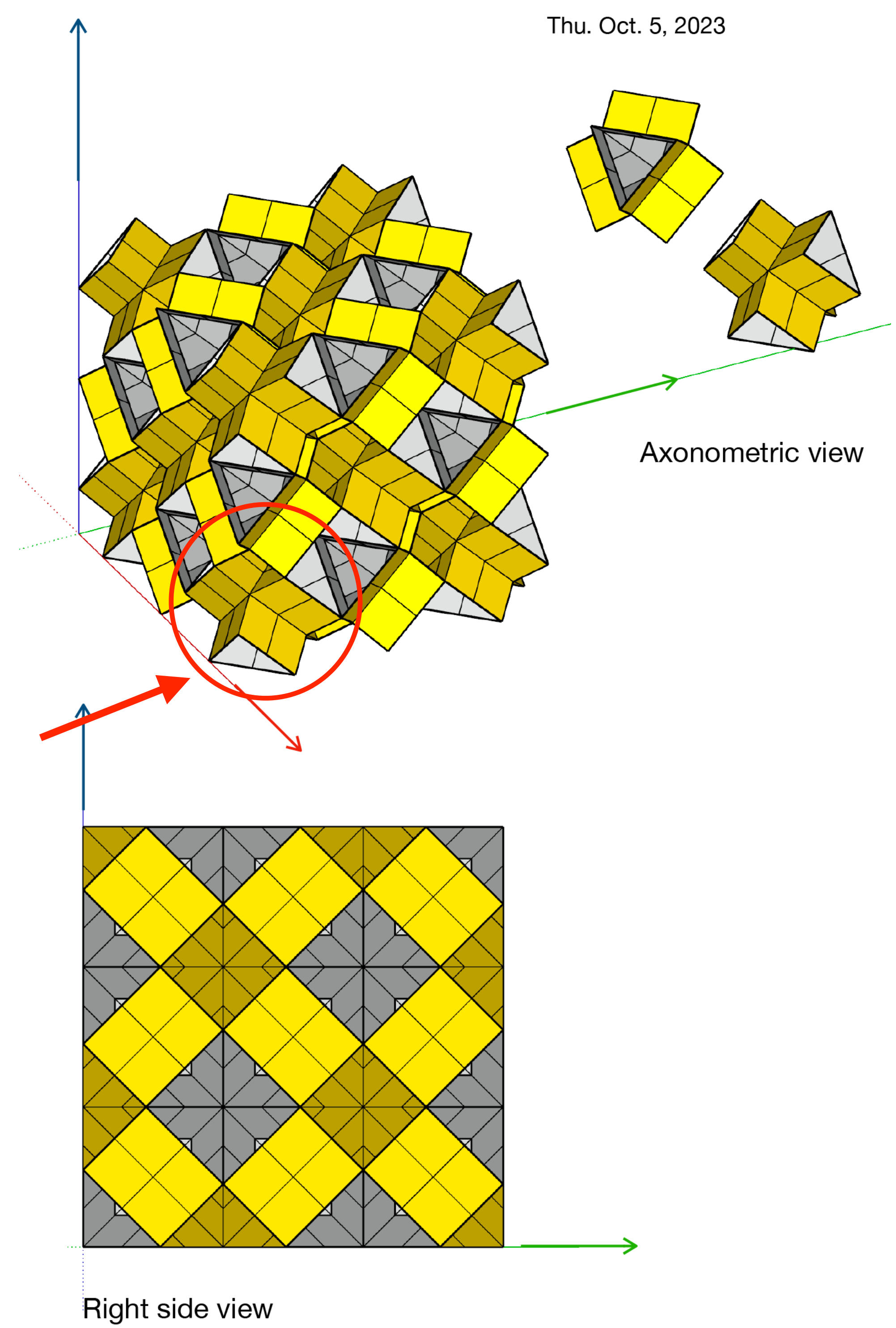


STEP 2: HALVE THE EXTRUSION LENGTH



STEP 3: "PUSH-FIT" THE EXTRUDED PRISMS



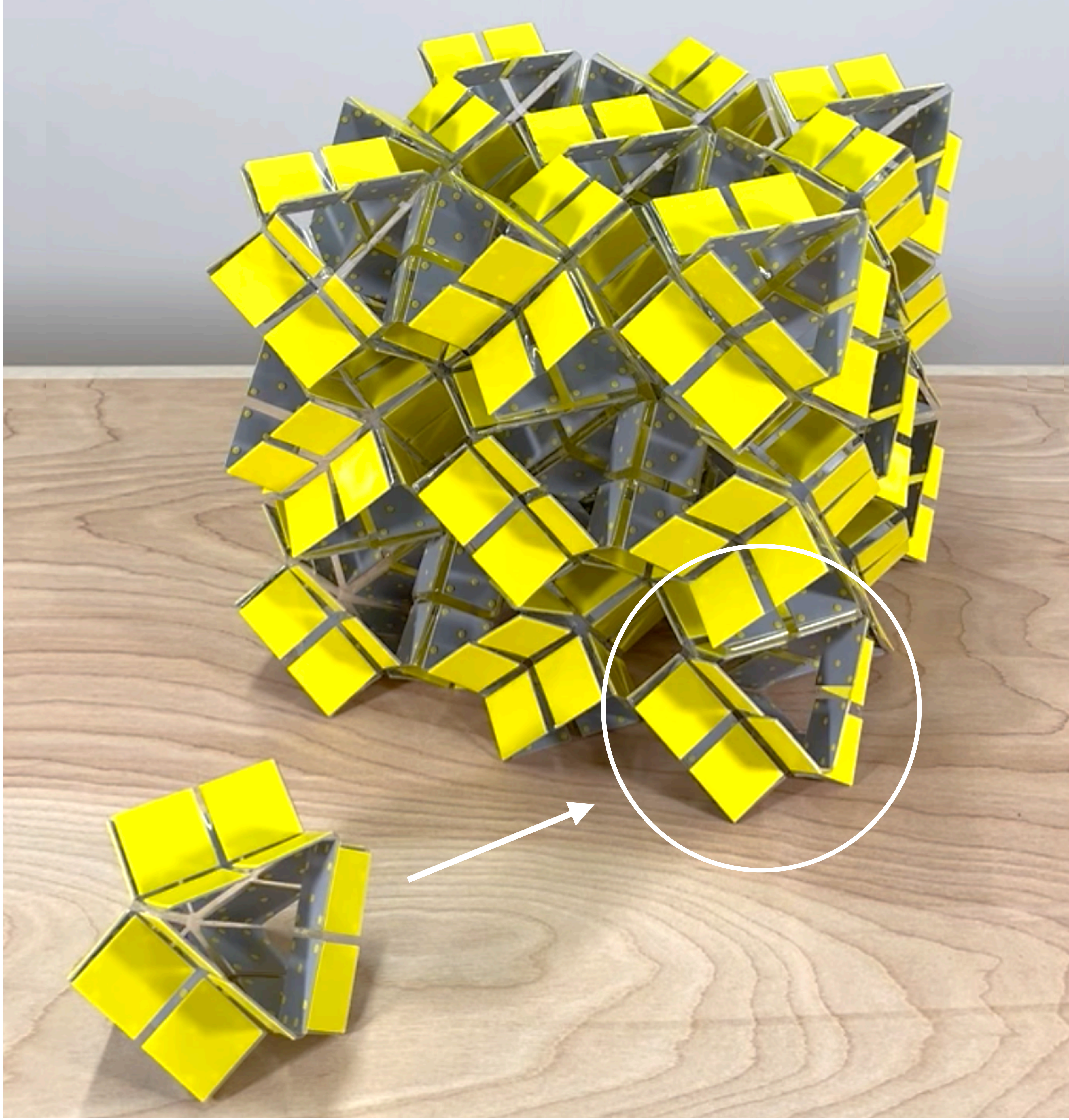


MATERIAL #1

TETRAHEDRA & OCTAHEDRA

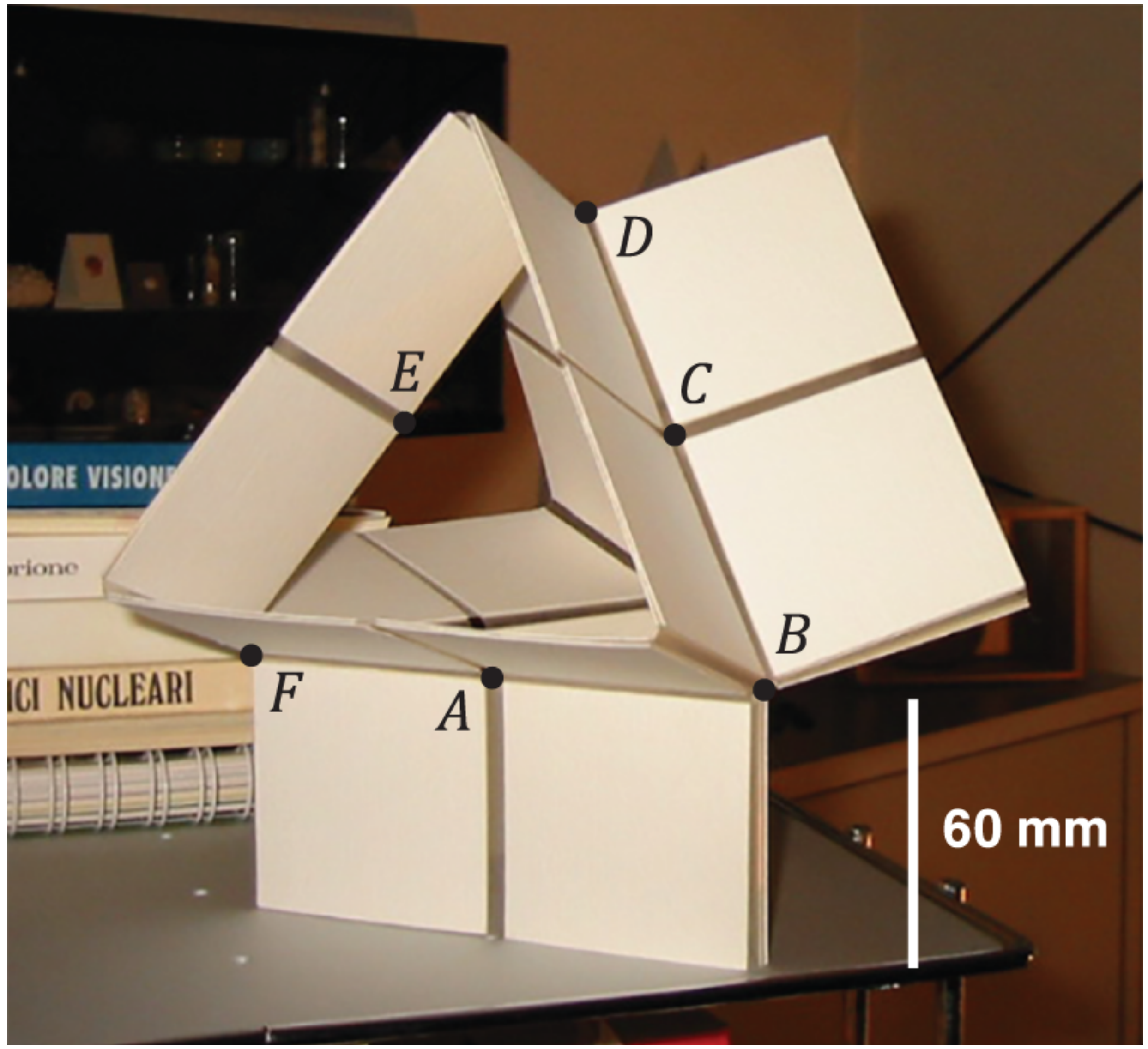
(FLAT-FOLDABLE)

MATERIAL #1
PHYSICAL MODEL



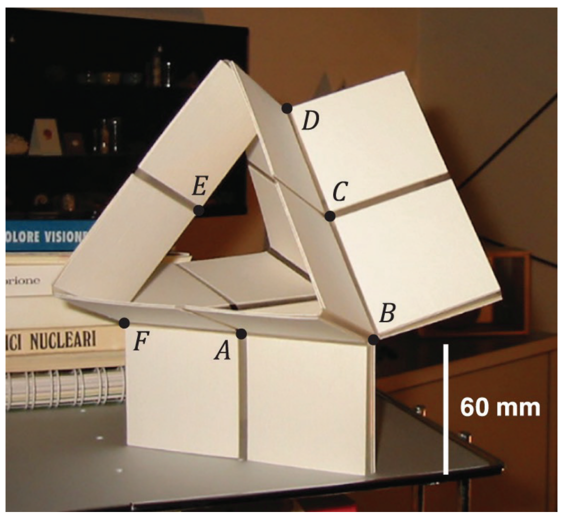
(Model fabrication:
E. Montano, J. Minnick,
D. Munechica, J. Cuellar,
P. Trogu, 2023)

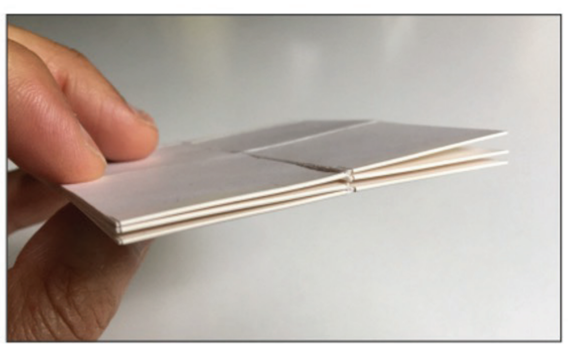
EXTRUDED TETRAHEDRON: FROM NON RIGIDLY FLAT-FOLDABLE TO RIGIDLY FLAT-FOLDABLE.



Original 1996 prismatic tetrahedron model with six representative vertices.

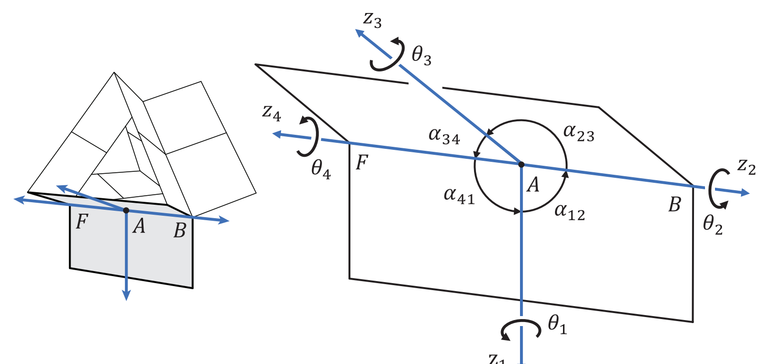
Scarpa's transformable model (1996)



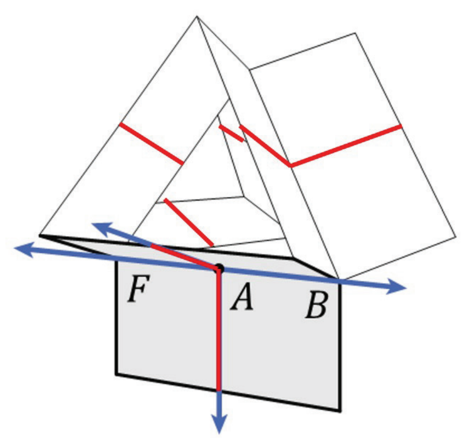


It is flat-foldable

Rigid foldability analysis



D-H notation of the four-crease vertex *A*.



inactive creases

It is not rigidly foldable

Conditions:

$$\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = \frac{\pi}{2}$$

Closure equation (D-H):

$$\boldsymbol{Q}_{21}\boldsymbol{Q}_{32} = \boldsymbol{Q}_{41}\boldsymbol{Q}_{34}$$

where

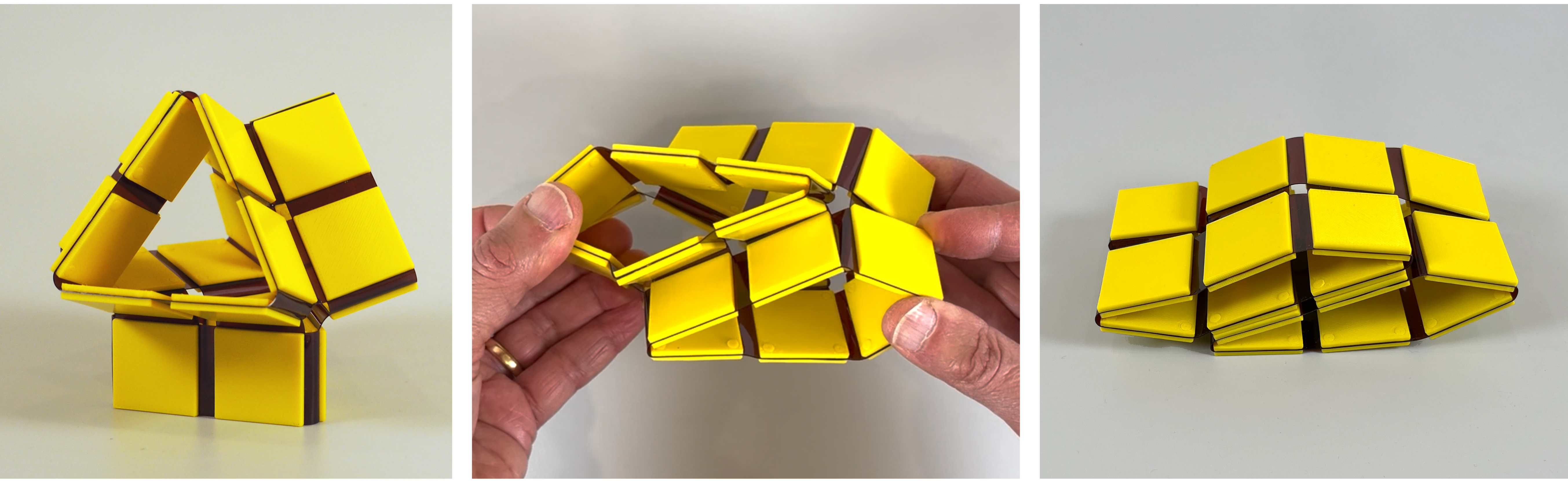
$$\boldsymbol{Q}_{(i+1)i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_{i(i+1)}\sin\theta_i & \sin\alpha_{i(i+1)}\sin\theta_i \\ \sin\theta_i & \cos\alpha_{i(i+1)}\cos\theta_i & -\sin\alpha_{i(i+1)}\cos\theta_i \\ 0 & \sin\alpha_{i(i+1)} & \cos\alpha_{i(i+1)} \end{bmatrix}$$

$$\boldsymbol{Q}_{i(i+1)} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\cos\alpha_{i(i+1)}\sin\theta_i & \cos\alpha_{i(i+1)}\cos\theta_i & \sin\alpha_{i(i+1)} \\ \sin\alpha_{i(i+1)}\sin\theta_i & -\sin\alpha_{i(i+1)}\cos\theta_i & \cos\alpha_{i(i+1)} \end{bmatrix}$$

Solution:

$\theta_1 = \theta_3 = 0,\ \theta_2 = \theta_4;$
or $\theta_2 = \theta_4 = 0,\ \theta_1 = \theta_3.$

SCARPA'S ORIGINAL SHAPE: NON-RIGIDLY FOLDABLE

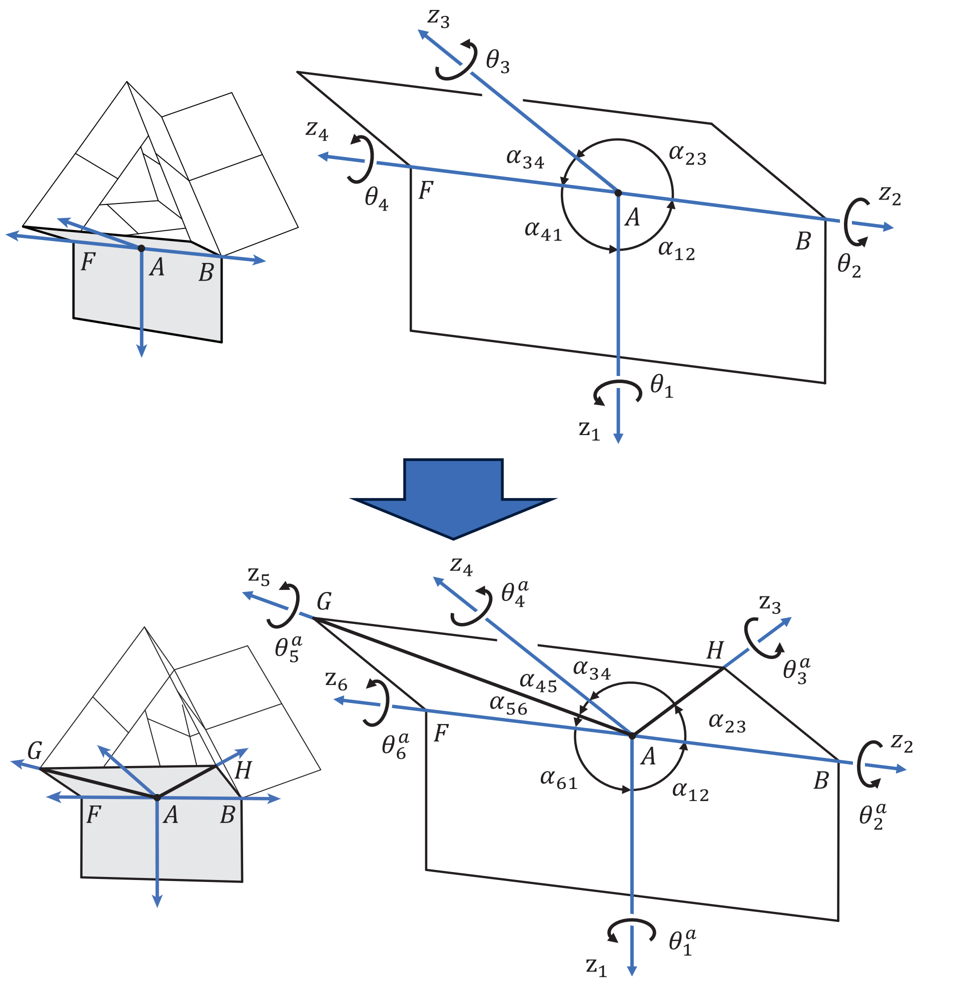


a b c

Non-rigid folding of prismatic tetrahedron: (a) Stable configuration; (b) Partially deformed configuration; (c) Flat-folded configuration.

(Model fabrication: H. Feng, W. Shi, P. Trogu, E. Montano, J. Minnick, G. Montalvo, T. Casanova, 2023)

Step 1: add two creases at vertex *A*



Altered vertex *A* with two additional symmetric creases.

Kinematics of the altered six-crease vertex

Conditions:

$$\alpha_{12} = \alpha_{61} = \frac{\pi}{2},$$

$$\alpha_{23} = \alpha_{34} = \alpha_{45} = \alpha_{56} = \frac{\pi}{4},$$

$$\theta_2^a = \theta_6^a, \theta_3^a = \theta_5^a.$$

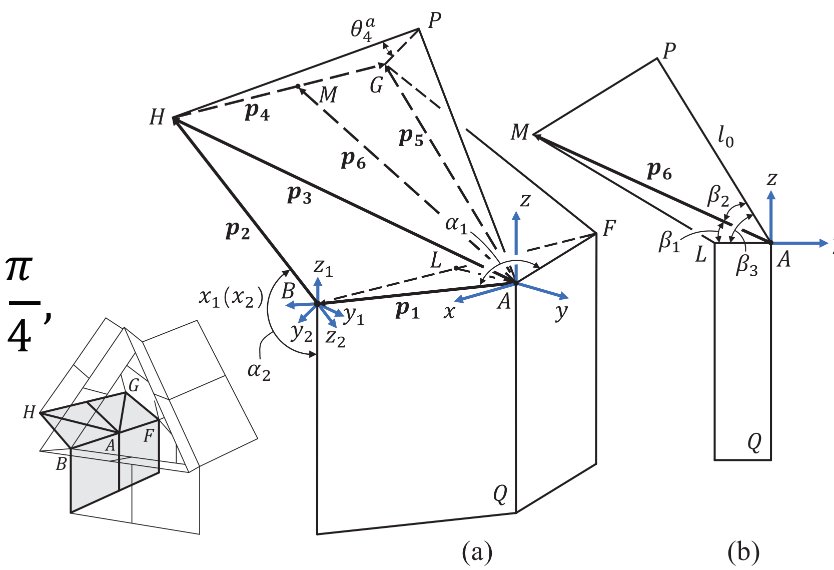
Assuming:

$$\theta_1^a = \alpha_1, \theta_2^a = \alpha_2.$$

Solution:

$$\theta_4^a = \operatorname{acos}\left(1 - 2\left(\sin\left(\frac{\alpha_1}{2}\right) - \cos\left(\frac{\alpha_1}{2}\right)\sin(\alpha_2)\right)^2\right);$$

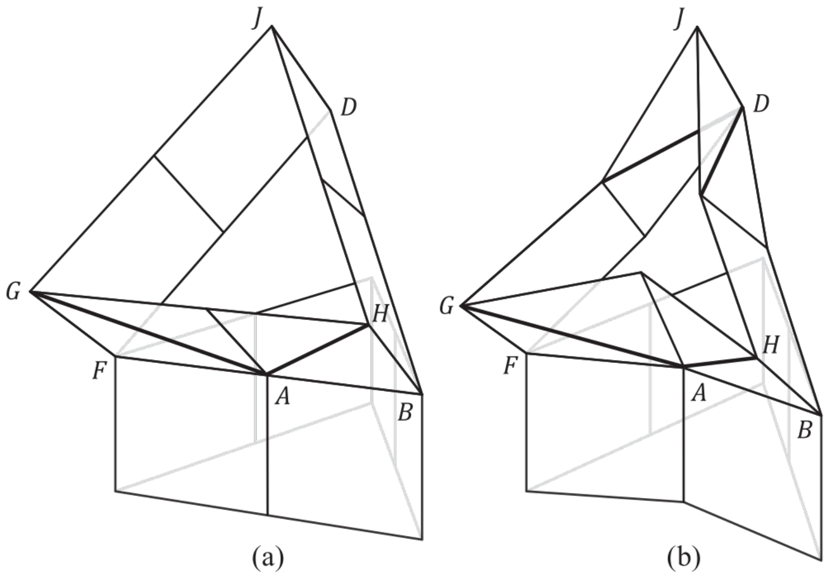
$$\theta_5^a = f(\alpha_1, \alpha_2).$$



(a) Coordinate system setup; (b) Side view from the y-A-z plane.

The altered six-crease vertex *A* has 2 Dof

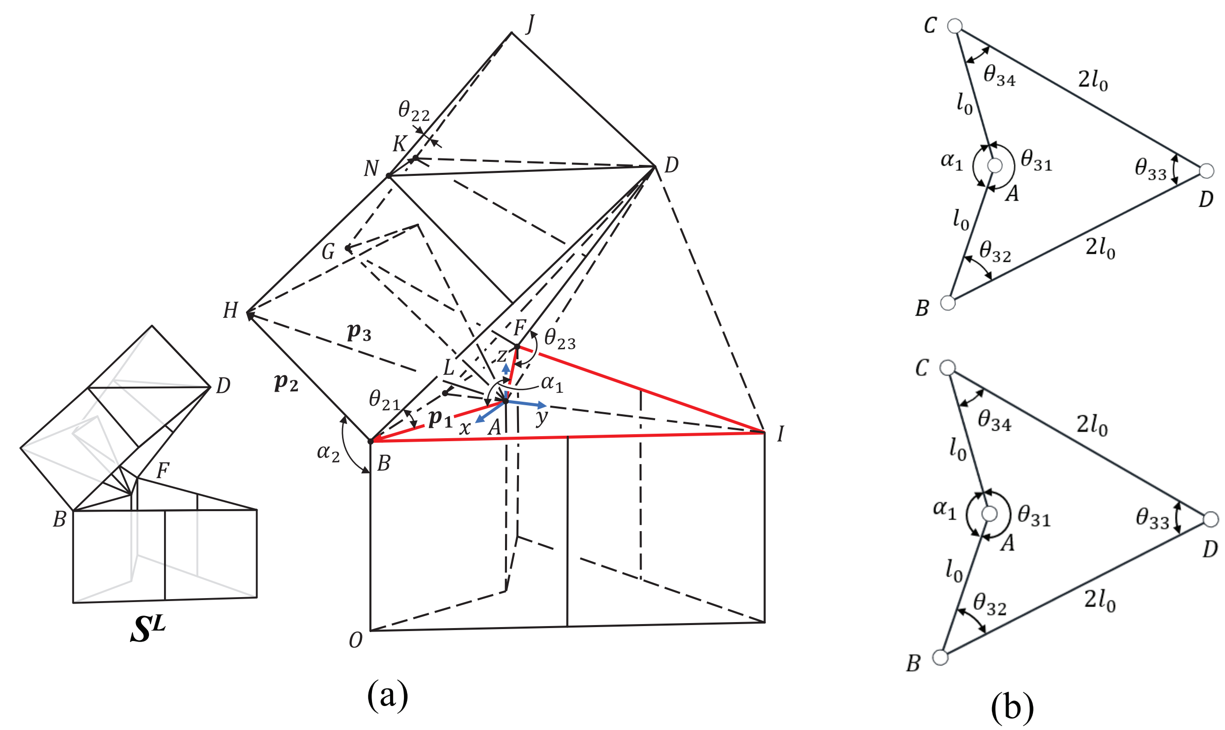
Step 2: add two creases at vertex D



Double triangular prism group with (a) only one altered vertex A; (b) altered central vertex A and corner vertex D.

The creases AG and AH added in step 1 are inactive in (a).

Kinematics of double triangular prism group

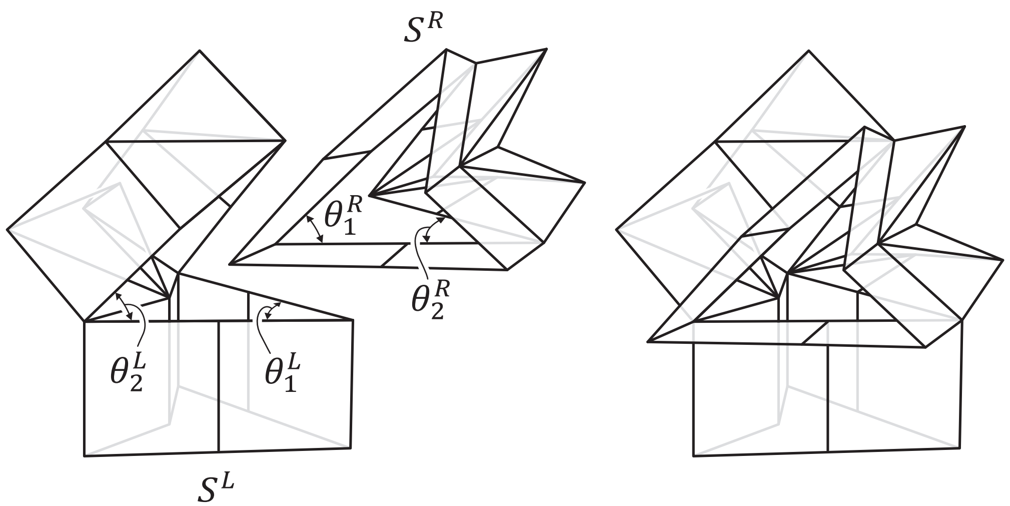


(a) Schematic diagram of two triangular prisms; (b) projection of the lower triangular prism along the line BO.

$$\alpha_1, \alpha_2 \rightarrow \begin{cases} \theta_3^a, \theta_4^a, \theta_5^a, \theta_6^a, \\ \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \\ \theta_{21}, \theta_{22}, \theta_{23}. \end{cases}$$

Double triangular prism group has 2 Dof

Step 3: compatibility analysis

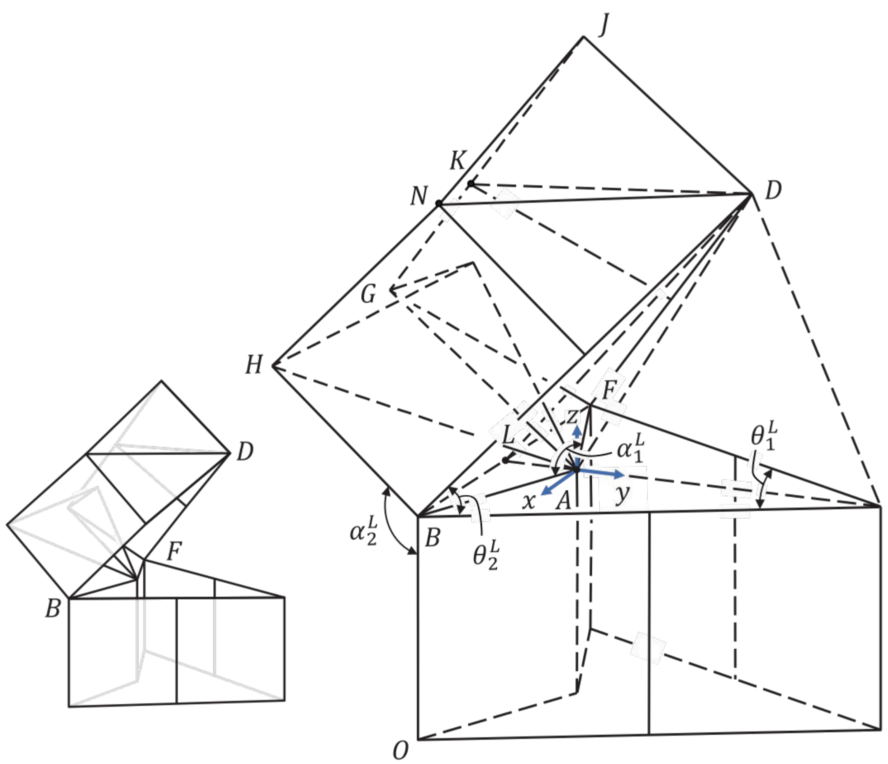


The two groups in exploded and assembled views

The compatible conditions of the whole structure are that the size lengths of the two groups are equal and

$$\theta_1^L = \theta_2^R, \theta_2^L = \theta_1^R.$$

Kinematics of the assembly angles



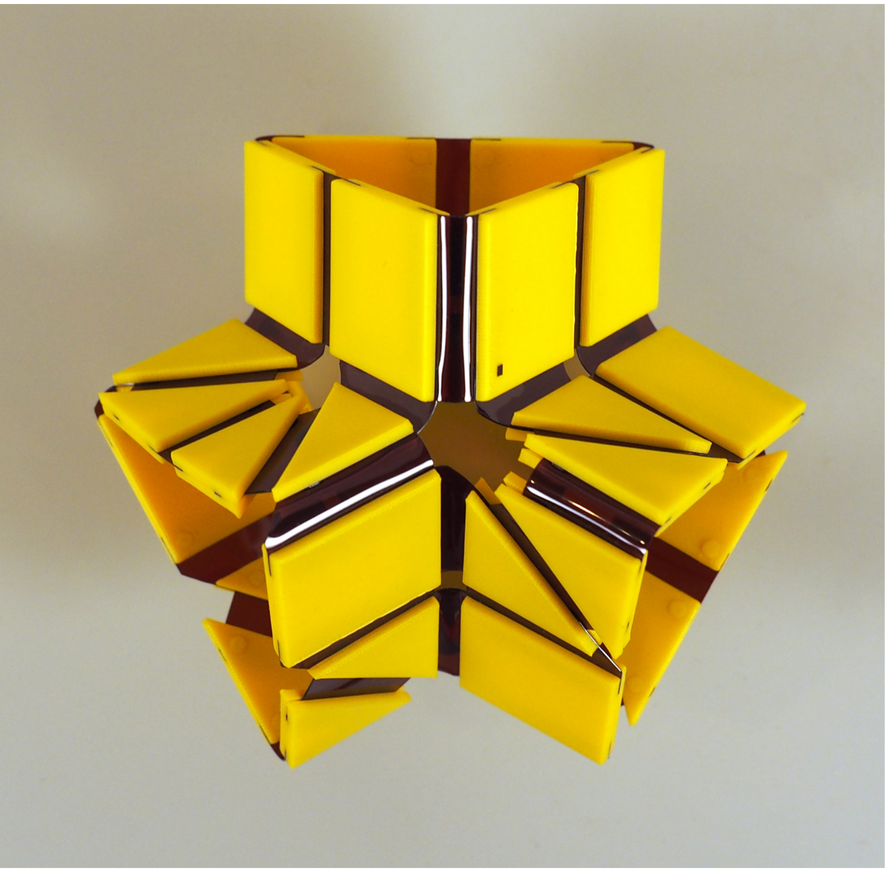
Relationship between the configuration angles and assembly angles.

Solution:

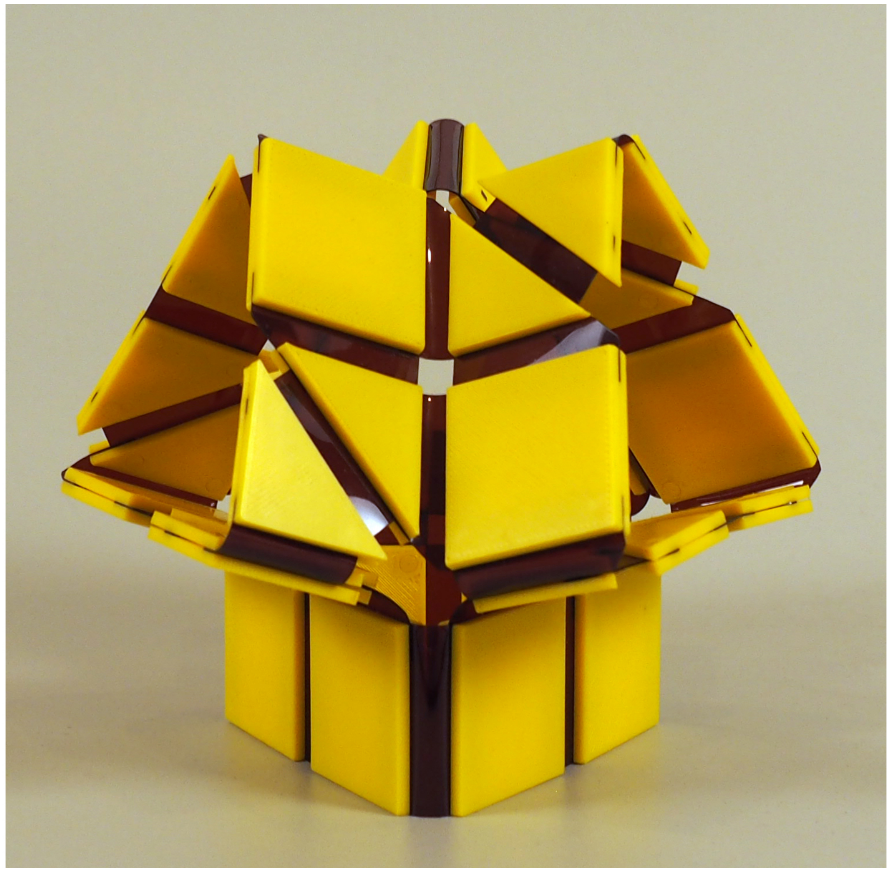
$$\theta_1^L = f(\alpha_1),$$

$$\theta_2^L = f(\alpha_1, \alpha_2).$$

MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES



Top view



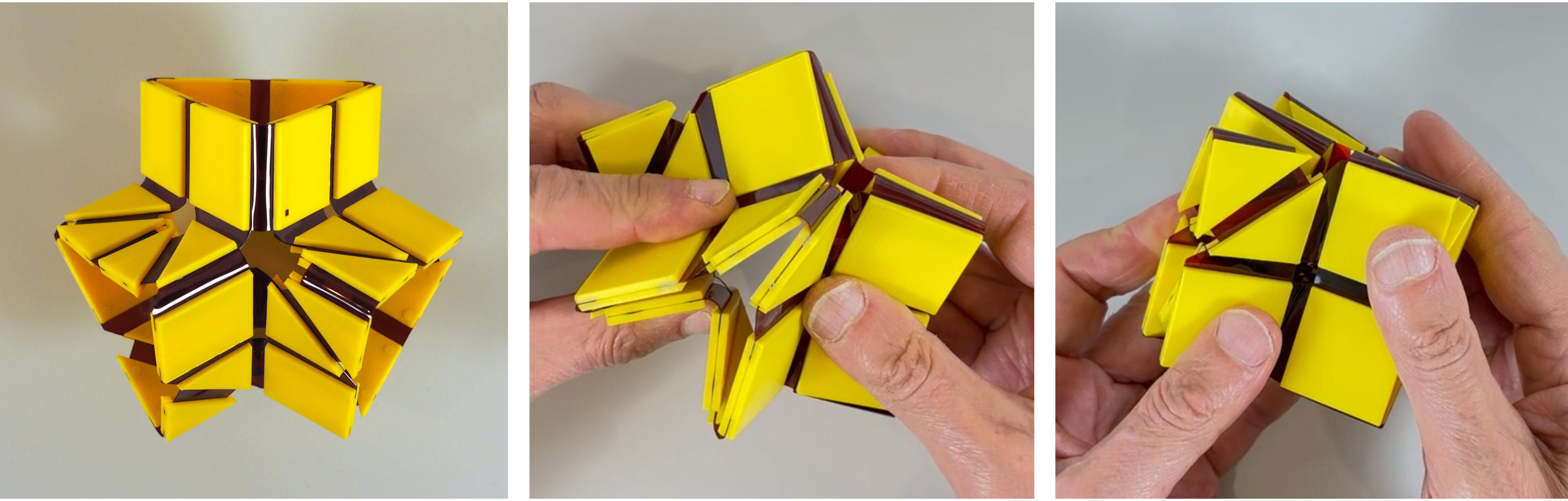
Front view

Right-side view

Orthographic views of the altered auxetic metamaterial.

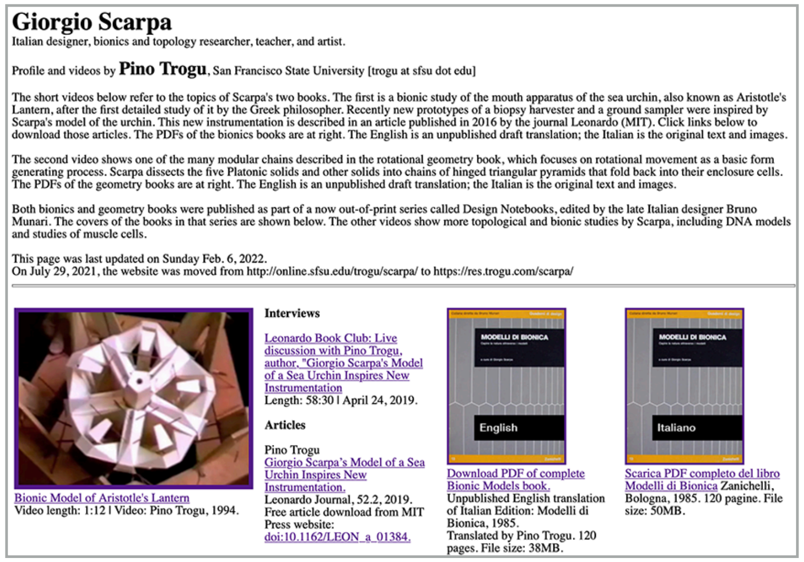
(Model fabrication: H. Feng, W. Shi, P. Trogu, E. Montano, J. Minnick, G. Montalvo, T. Casanova, 2023)

MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES

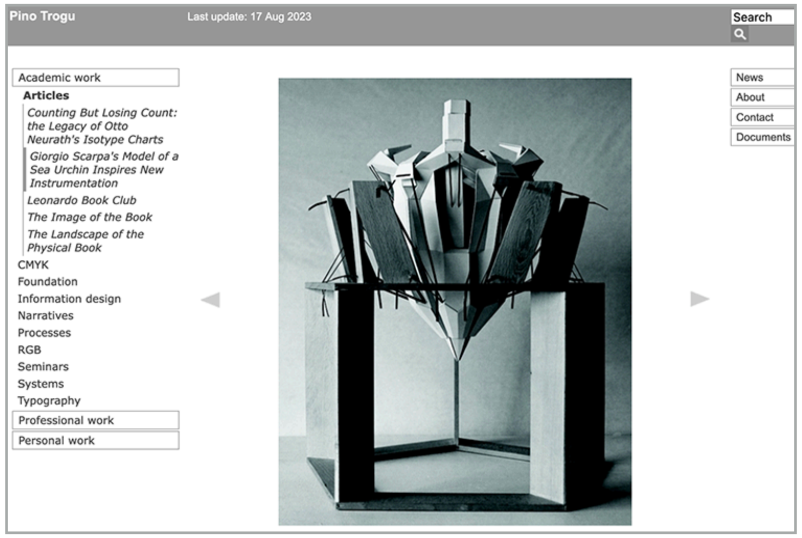


Rigid folding process of the altered auxetic metamaterial.

LINKS



res.trogu.com/scarpa



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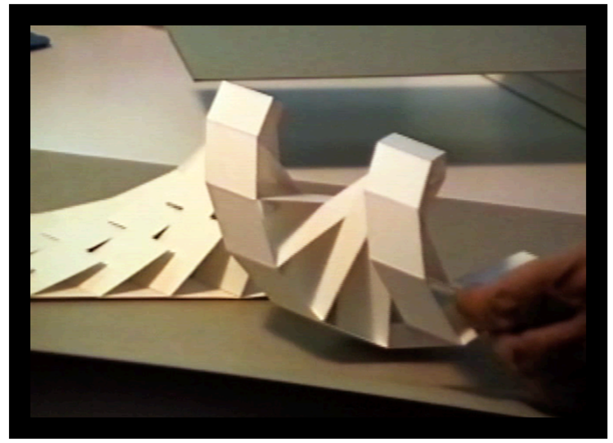
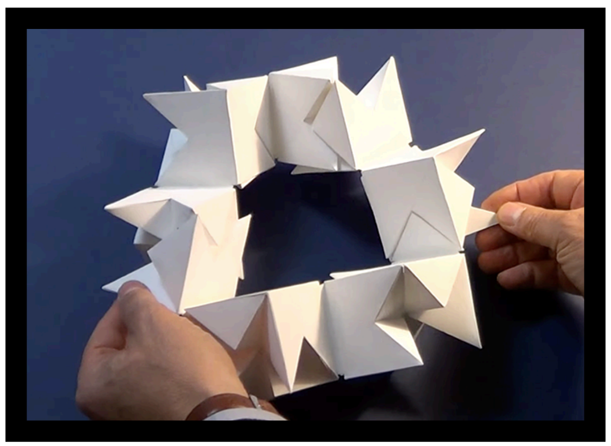
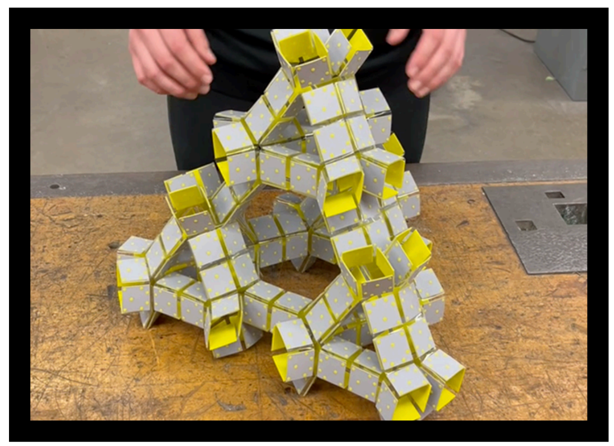
[Scarpa: Aristotle's Lantern]



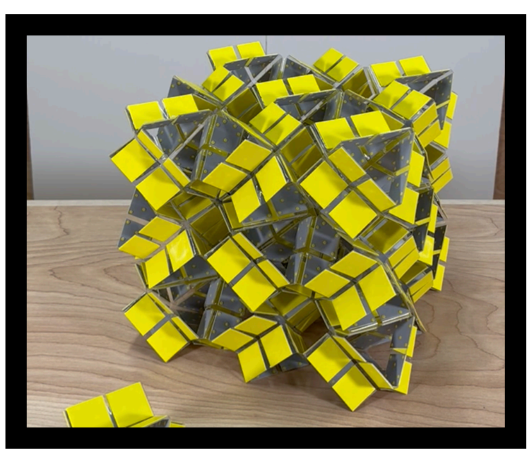
[Scarpa: DNA Model]



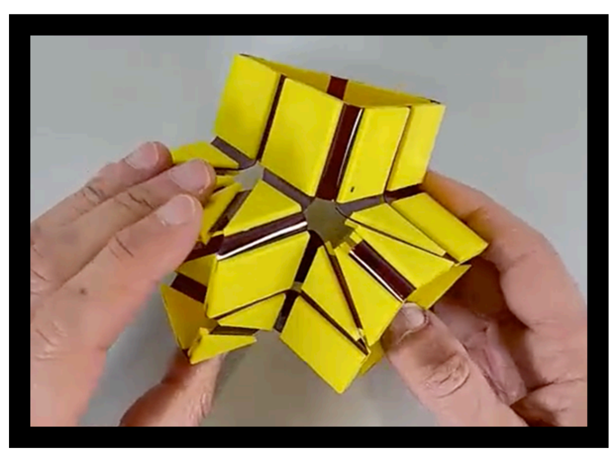
[Scarpa: Hexahedral Chain]



[Trogu: Material #6]



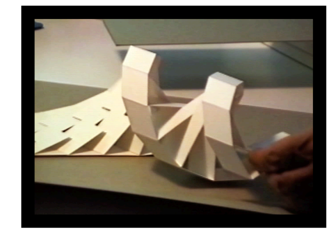
[Trogu: Material #1]



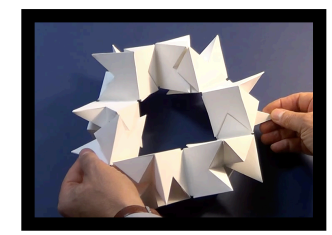
[Feng, Shi, Trogu, & Dai: Auxetic Metamaterial]

VIDEOS (online/local)

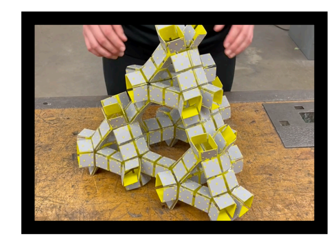
Scarpa: Aristotle's Lantern



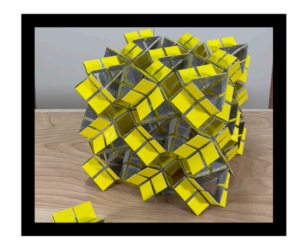
Scarpa: DNA Model



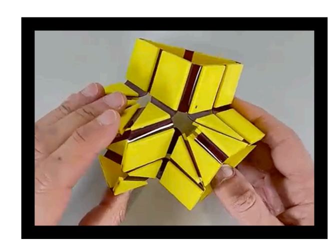
Scarpa: Hexahedral Chain



Trogu: Material #6



Trogu: Material #1

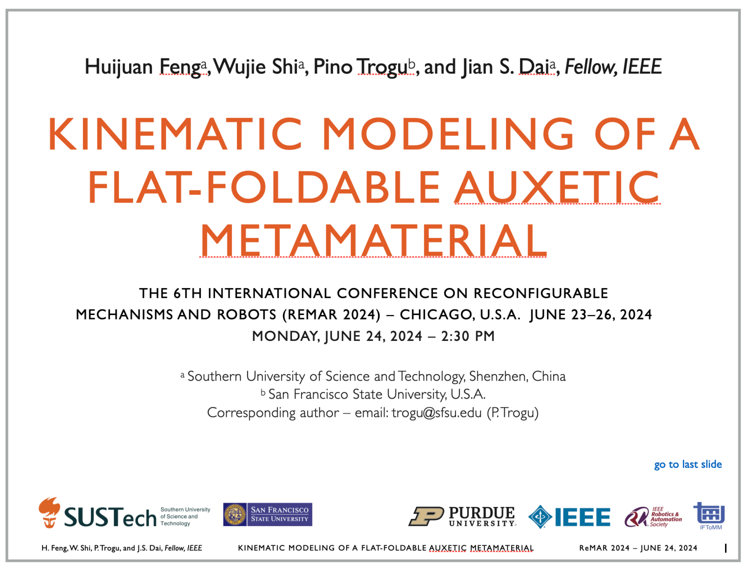


Feng, Shi, Trogu, & Dai: Auxetic Metamaterial

THANK YOU!

THE 6TH INTERNATIONAL CONFERENCE ON RECONFIGURABLE

MECHANISMS AND ROBOTS (REMAR 2024) – CHICAGO, U.S.A. JUNE 23–26, 2024



[Presentation Slides – PDF 32 MB]

CONTACT: TROGU@SFSU.EDU

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