PINO TROGU – SAN FRANCISCO STATE UNIVERSITY, USA
VISITING SCHOLAR, SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY (SUSTech) 2024–2025, SHENZHEN, CHINA

DESIGN OF FLAT-FOLDABLE AUXETIC METAMATERIALS

2025 SPRING INTERNATIONAL WORKSHOP

CHINA–POLAND JOINT WORKSHOP ON ADVANCED LIGHTWEIGHT STRUCTURES AND ENERGY ABSORPTION

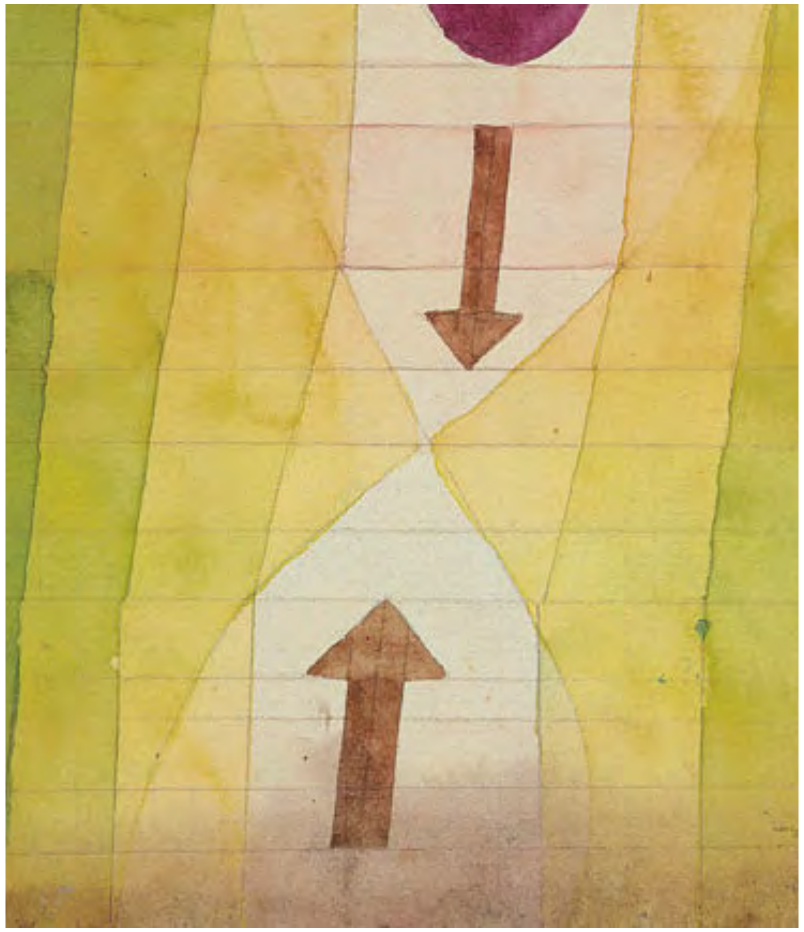
SOUTHEAST UNIVERSITY, NANJING — MAY 22, 2025, 11:20 AM

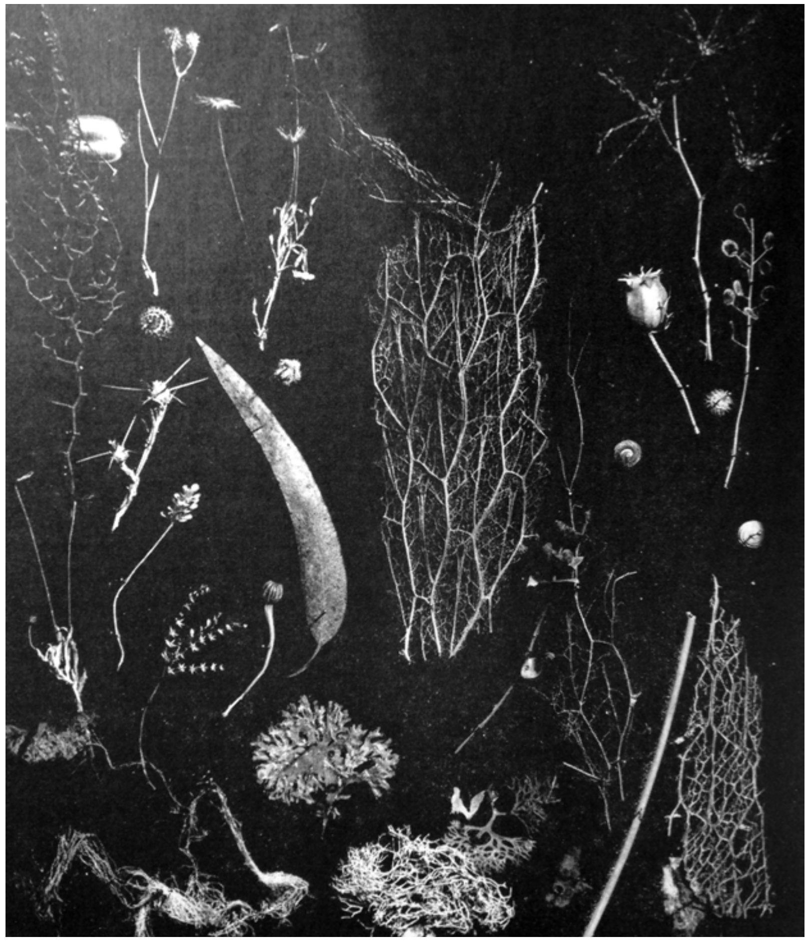
ROOM 304, CIVIL ENGINEERING BUILDING, JIULONGHU CAMPUS

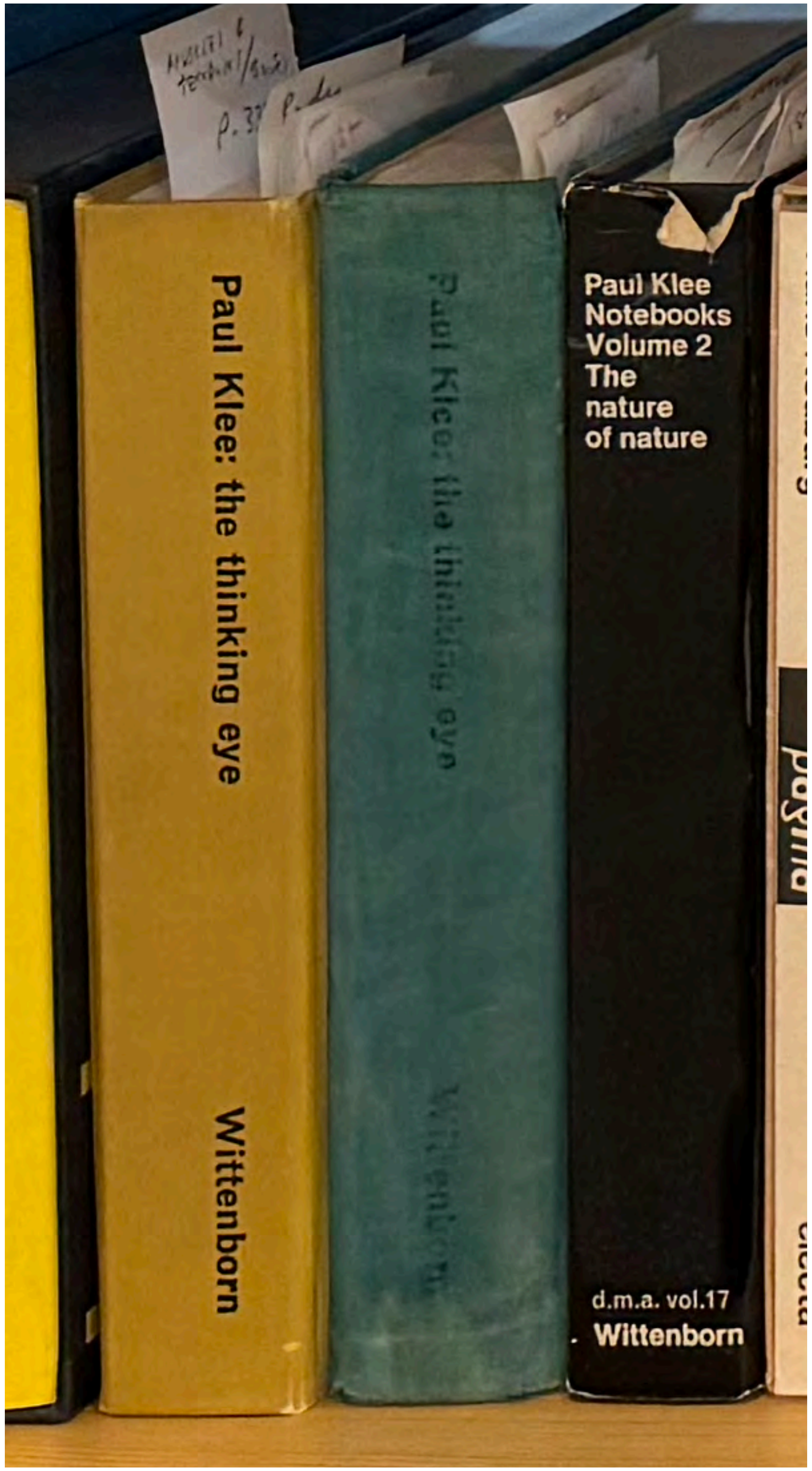
go to last slide

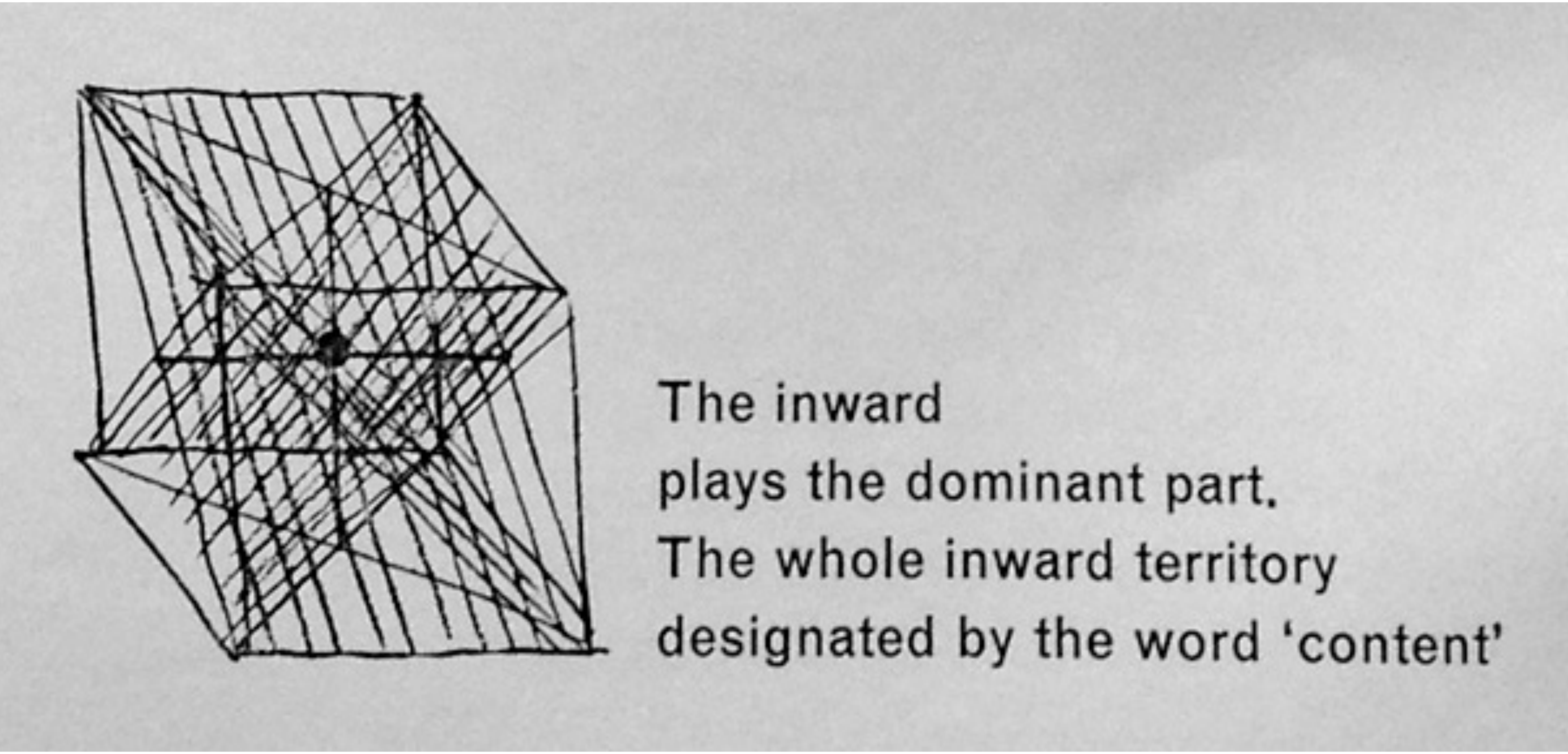
ORIGINS
AND INFLUENCES:
GIORGIO SCARPA

PAUL KLEE AT BAUHAUS: 1921–1931







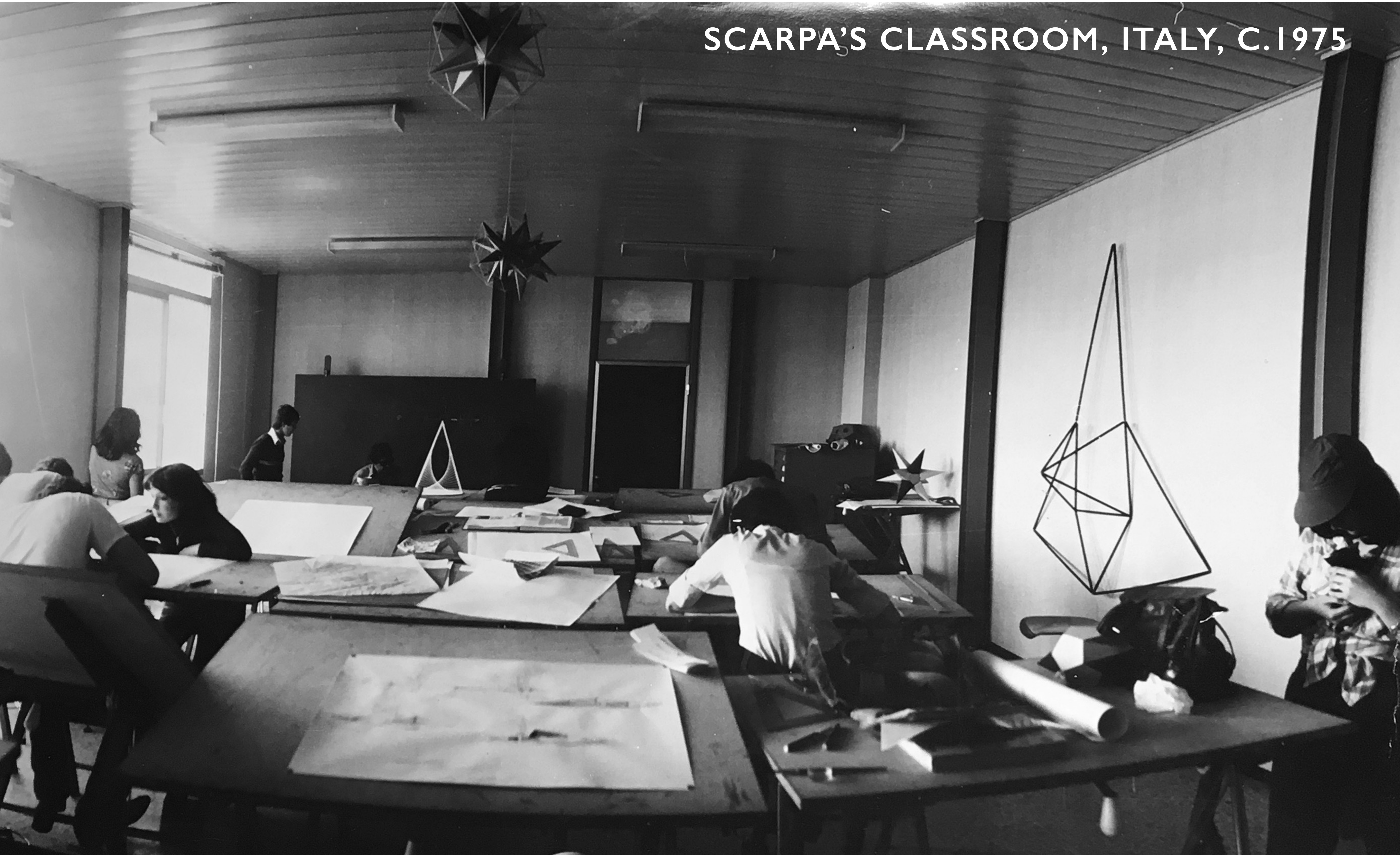


Paul Klee, *Notebooks Volume 1: The Thinking Eye*
New York: Wittenborn, 1961) p. 127

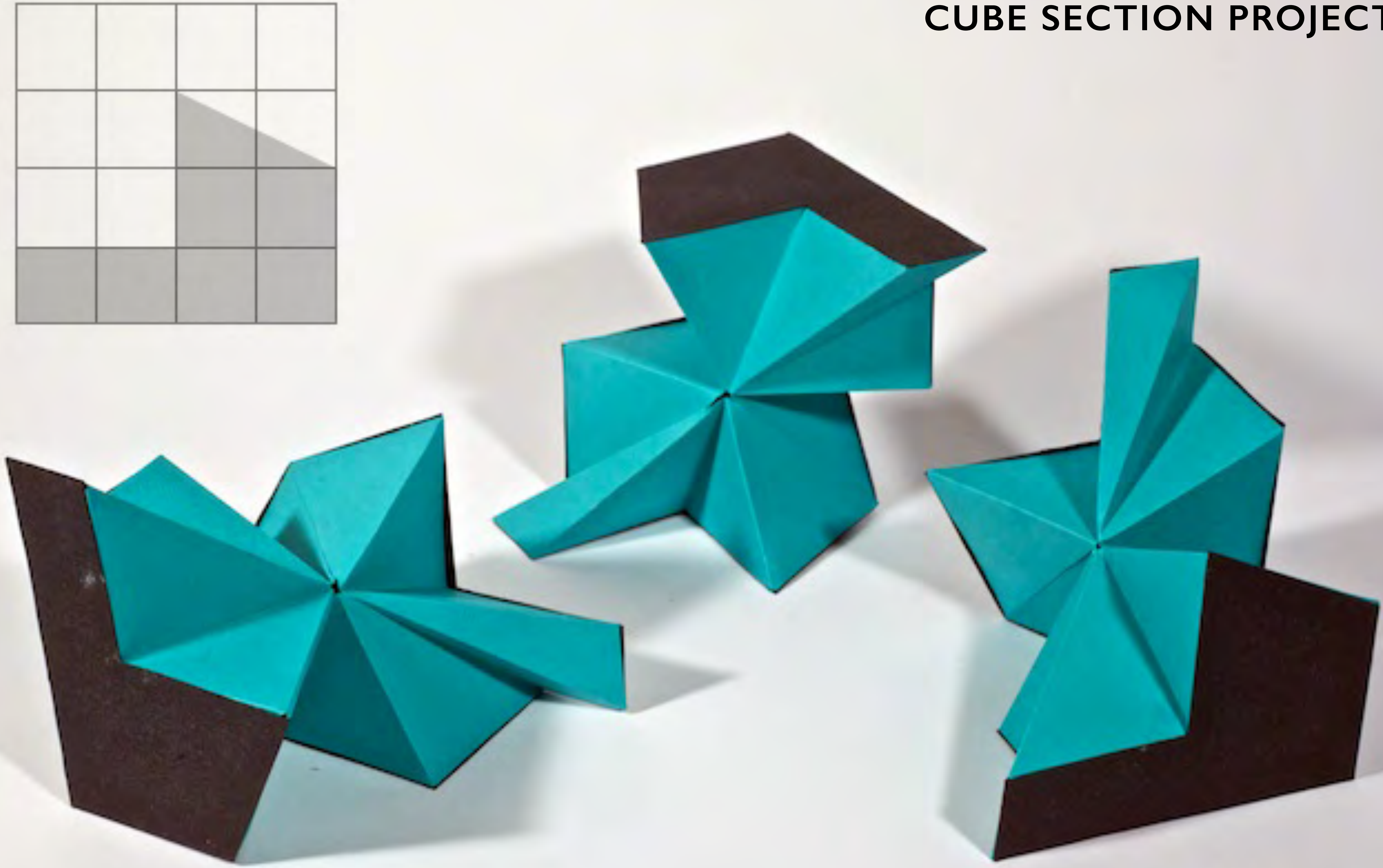
TEACHING:

TECHNICAL DRAWING & PERSPECTIVE.

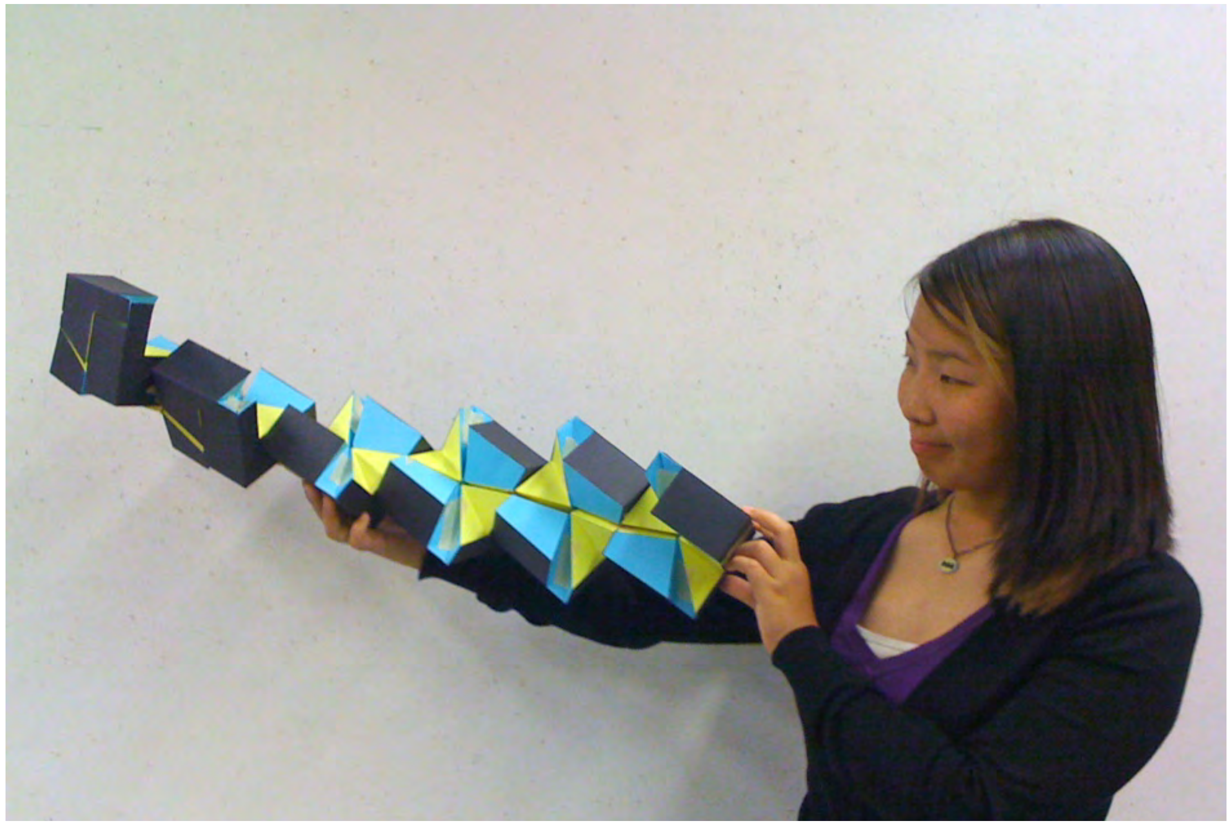
SCARPA’S CLASSROOM, ITALY, C.1975

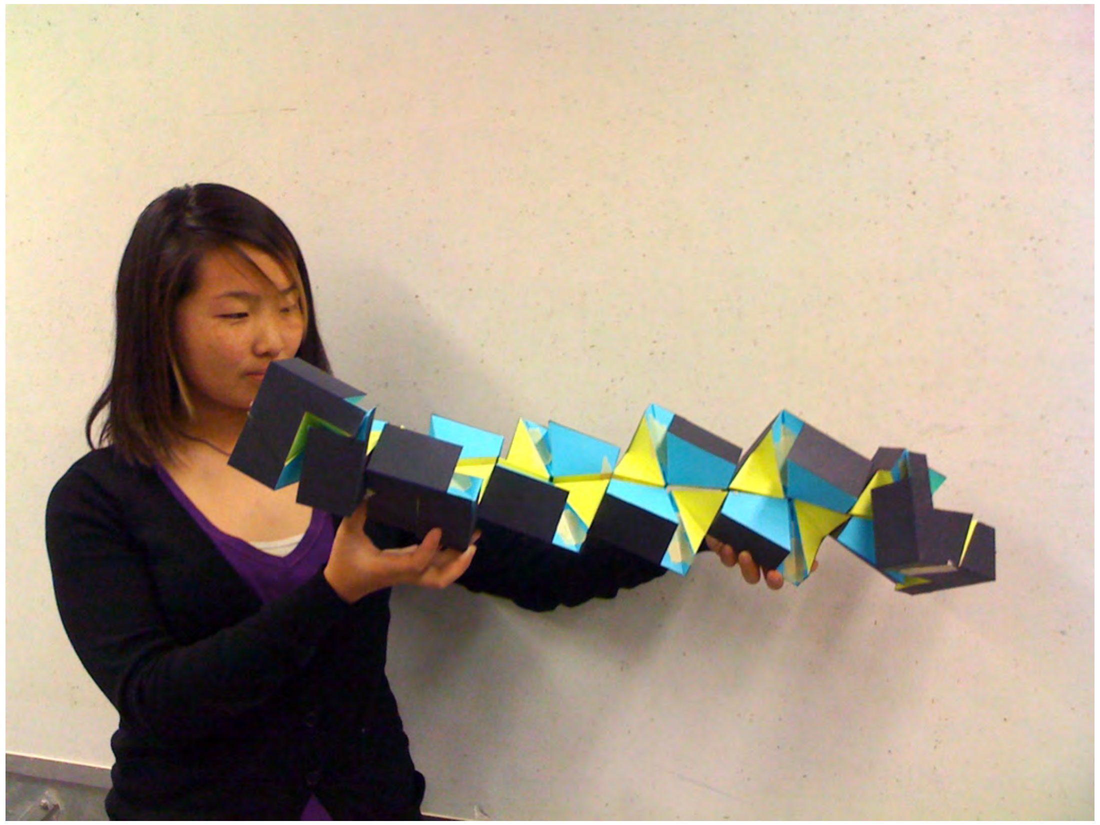


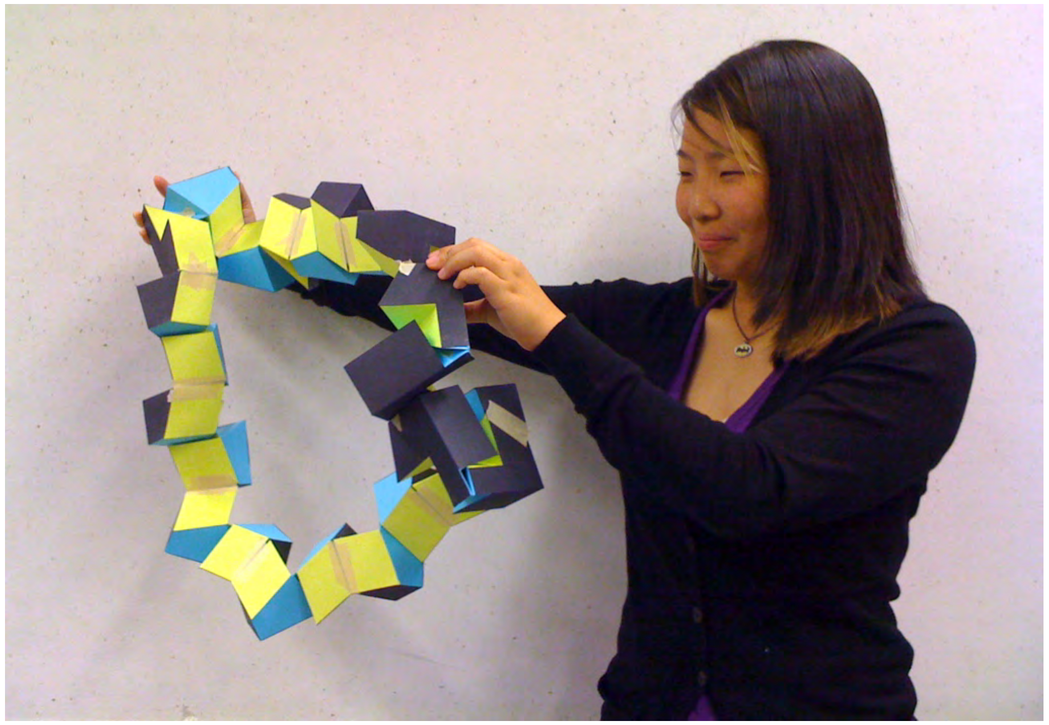
CUBE SECTION PROJECT



Trogu – Drafting & Sketching Course
San Francisco State university

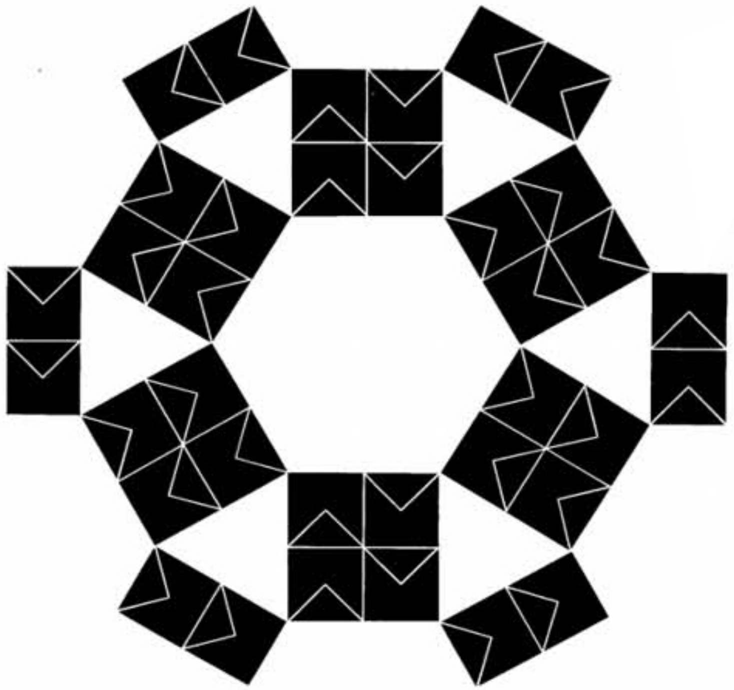


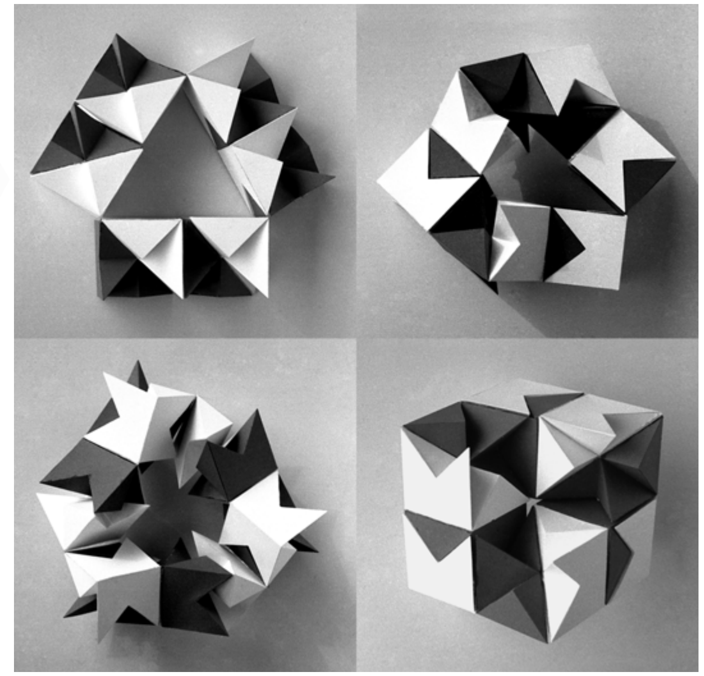


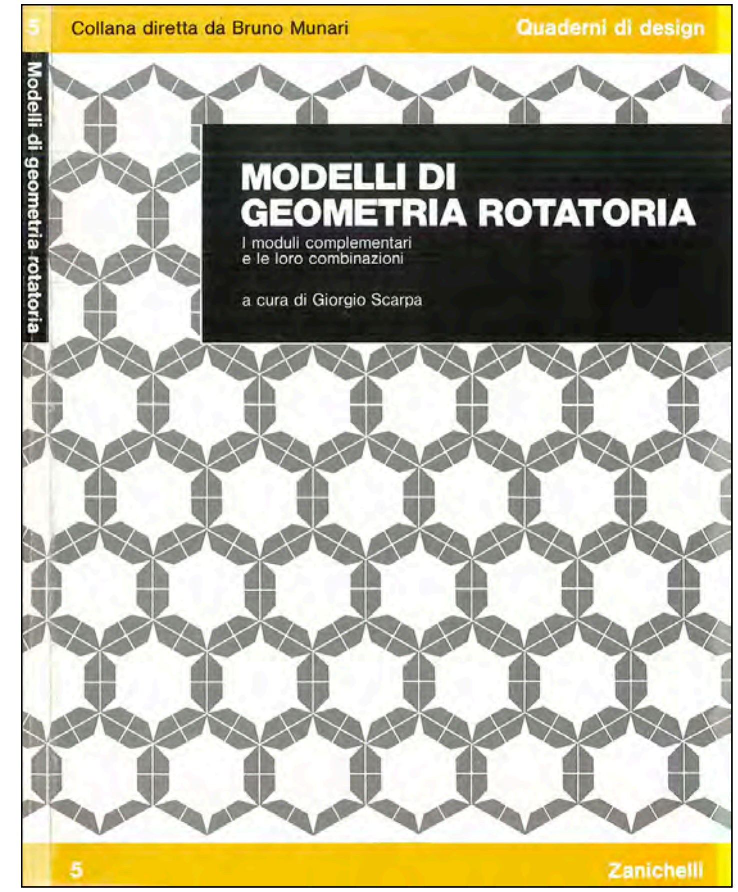


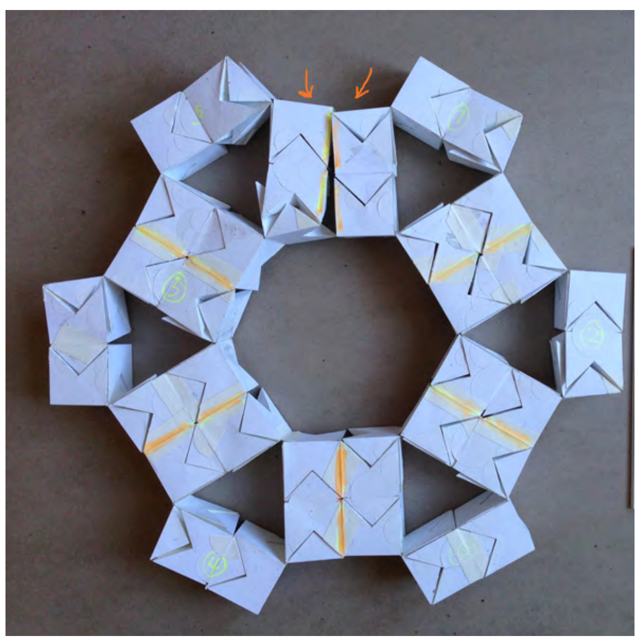
24-MODULE CLOSED CHAIN
FLORENCE YUEN
SAN FRANCISCO STATE UNIVERSITY
2012

SCARPA: MODELS OF ROTATIONAL GEOMETRY “TRANSFORMABLE FABRICS” 1978






5
Collana diretta da Bruno Munari
Quaderni di design
Modelli di geometria rotatoria
MODELLI DI
GEOMETRIA ROTATORIA
I moduli complementari
e le loro combinazioni
a cura di Giorgio Scarpa
5
Zanichelli

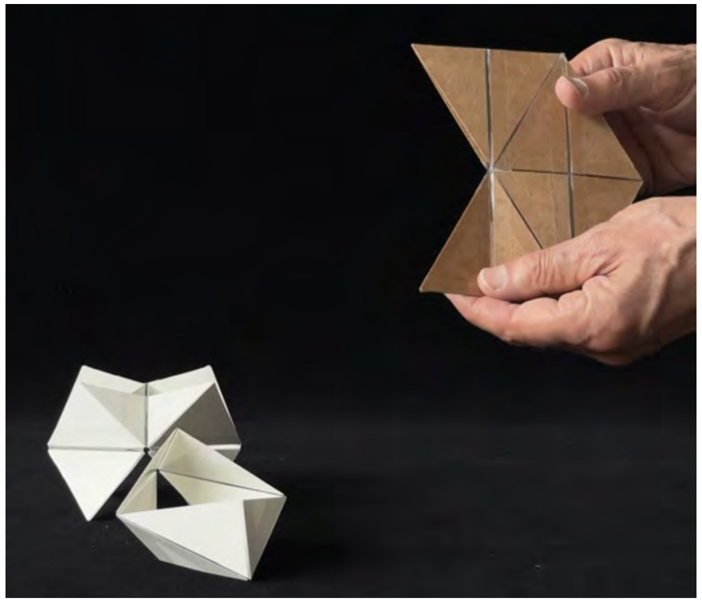


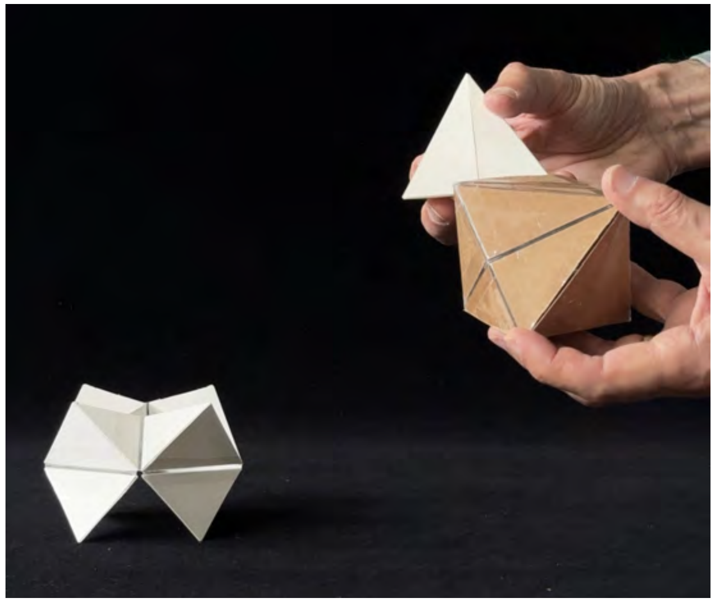
SCARPA TETRAHEDRAL STRIP, C. 1980
TROGU TETRAHEDRAL POD, 2025

Pod: housing for the tetrahedral strip composed of eight right-angle pyramids equal in volume to the octahedron with the same face as the housed tetrahedron

Strip: eight half triangles hinged together transform from a cubic shape to a regular tetrahedron.

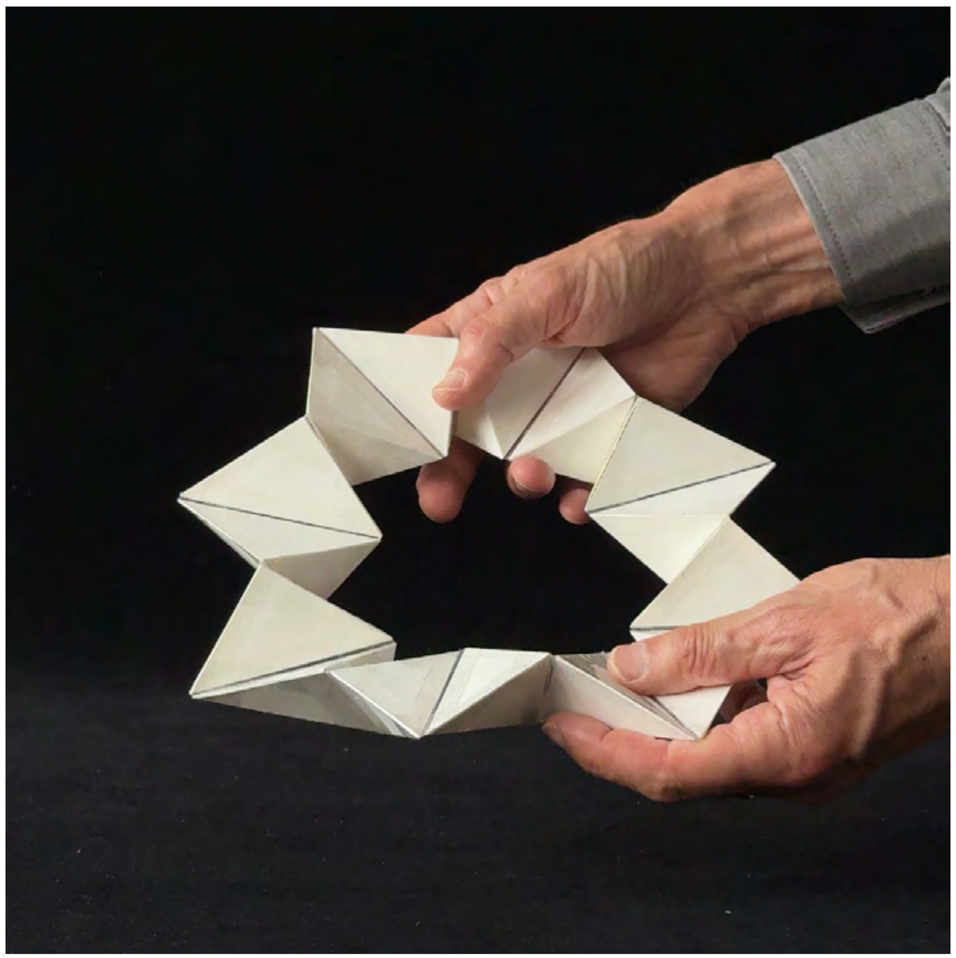
Octahedral strip transforms into an octahedron as well as a tetrahedron by the folding and doubling up of the faces.

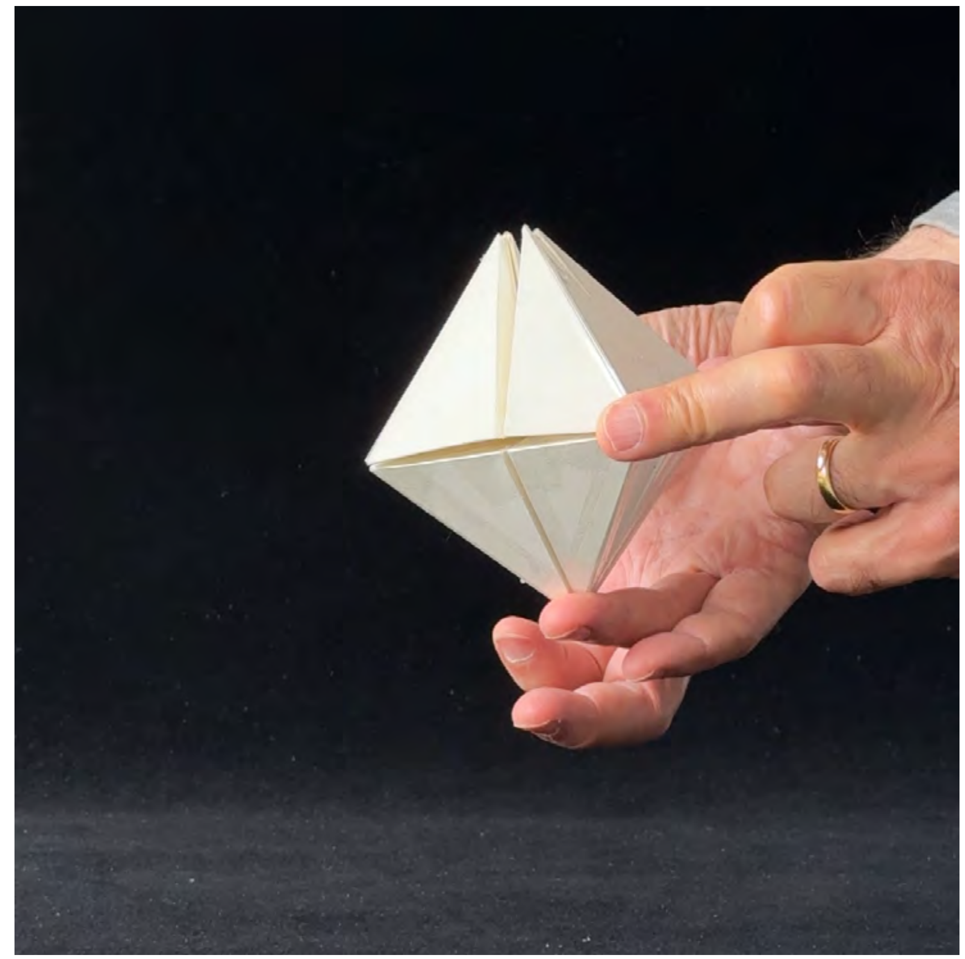




SCARPA OCTAHEDRAL CHAIN, 1978

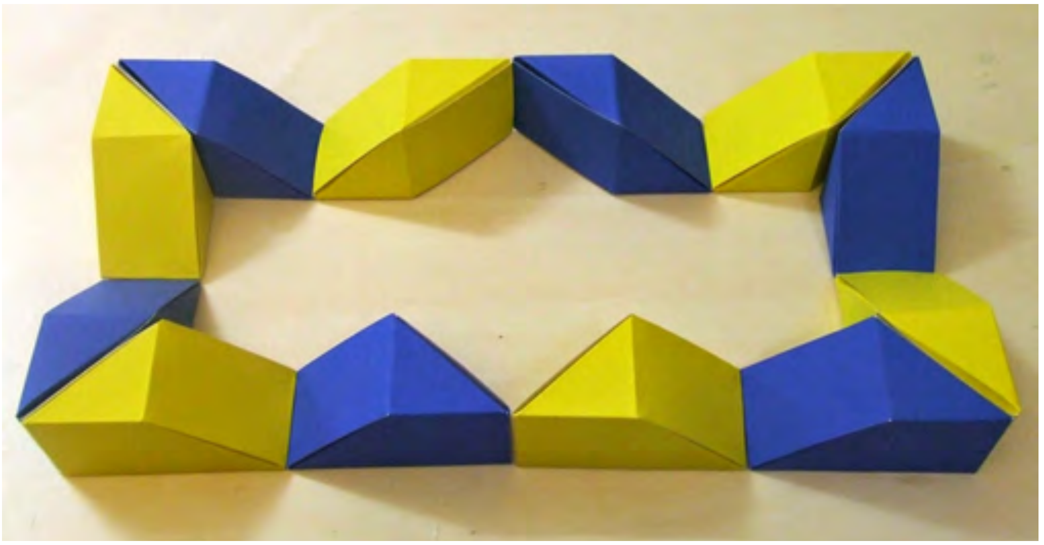
Chain: the same eight half triangles used in the tetrahedral pod, but with different hinge connecting, fold back into a regular octahedron.

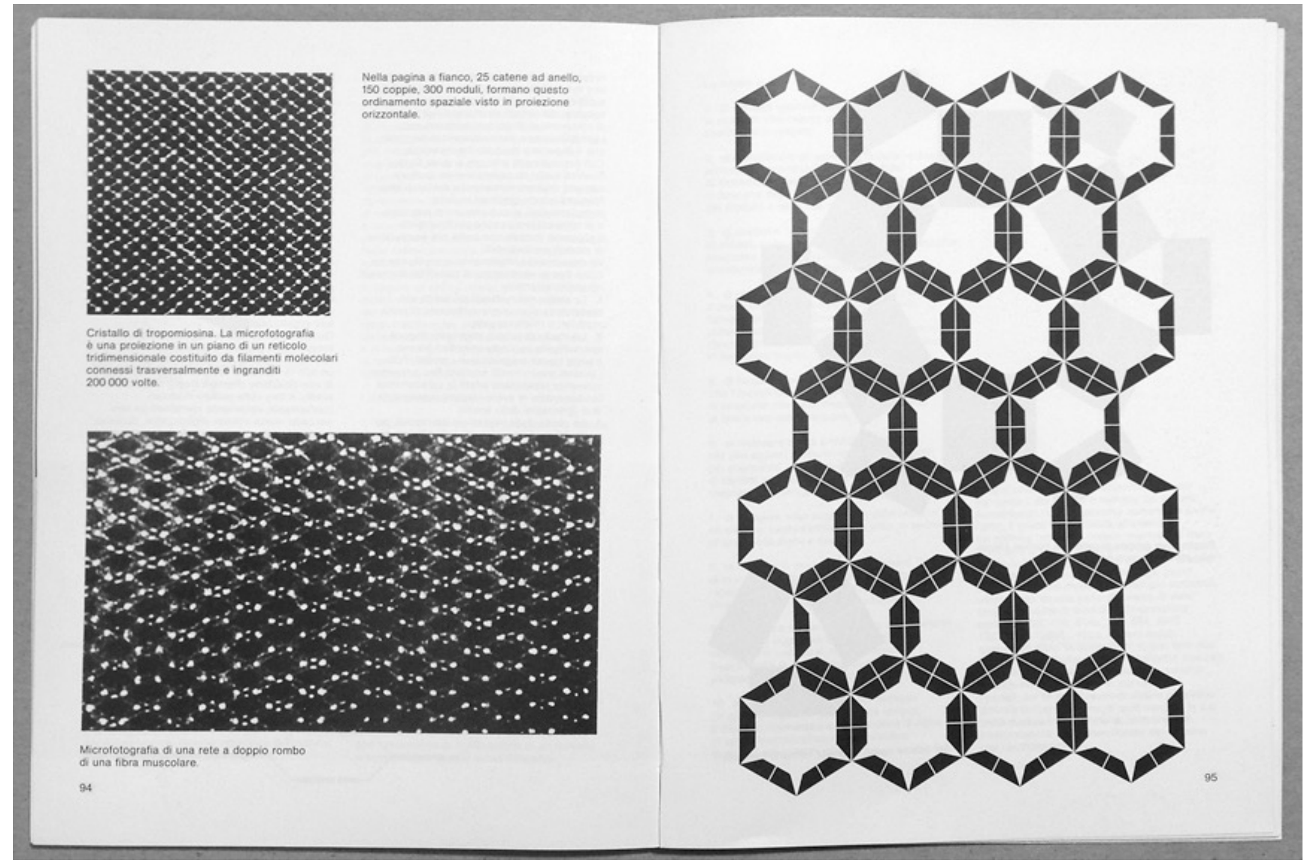




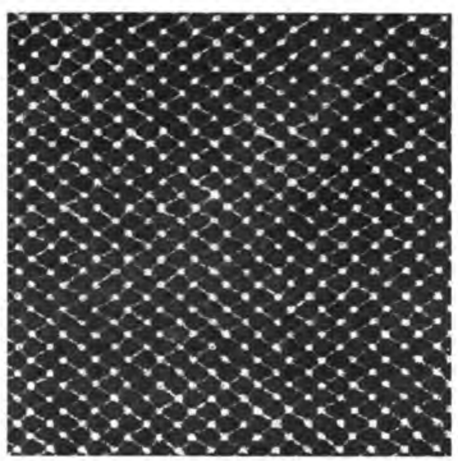
Replica by P.Trogu, 2025.

SCARPA TRANSFORMABLE FABRICS, 1978

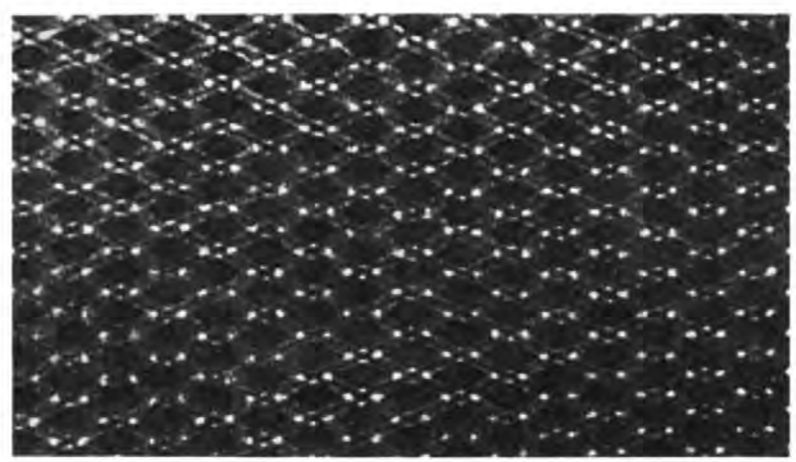




G. Scarpa, *Models of Rotational Geometry*, 1978



X-RAY PATTERN AND ELECTRON MICROGRAPH of tropomyosin crystals were interpreted together to decipher the structure and interactions of the molecule. The X-ray-diffraction pattern (*left*) could not be readily interpreted by itself but could be understood with the aid of an electron micrograph (*right*) of a negatively stained tropomyosin crystal. The micrograph is a projection in two dimensions of a three-dimensional lattice of cross-connected molecular strands enlarged 200,000 diameters. The strands are wavy (resembling a sine wave, the projection of a helix), so that their mesh forms kite-shaped regions. The repeat along a strand (a long arm plus a short arm) is about 400 angstroms. In parts of the micrograph one can see that strands are made up of two filaments.



DOUBLE-DIAMOND MESH is produced when the *TnT* subunit of troponin is crystallized with tropomyosin. The period along the strands is still 400 angstroms but the crossovers are separated to produce a mesh of large and small diamonds. Tropomyosin molecules cross at or near position of *TnT* (*bright nodes*), which apparently generates new cross-connections.

Carolyn Cohen, The Protein Switch of Muscle Contraction.
Scientific American 233, no. 5 (1975): pp 42, 44

“An experiment which is currently under way has for its aim the construction of **tranformable fabrics** with the following goals, in the order:

1 . The study of more appropriate materials for the construction of a relevant number of modular units and their respective connections.
2. The study of electronic tessellations spatially arranged in a binary order, with which we can magnetize the modules. All the tranformable models constructed so far present in fact the fundamental characteristic of having the connections (hinges) in two dimensions in space.
3. The study of the interactions of the modules, through mathematical models, to be fed to the computer, in such a way that it will be possible to rapidly evaluate various solutions and thus choose those that better adapt to the specific goals.
4. The study of diagrams to illustrate the order of each modular unit rotation in such a way that the set of specific series of rotations, and therefore the precise combinations of modules, can be visualized as shrinkings, stretchings, curlings, vibrations, etc. In this sense, the electronic paths in the tilings of modular connections contribute to structure, in a given model, the forms designated by the selected linguistic terms.
5. The study of how to interact with the articulated models through the use of electronic remote controls.”

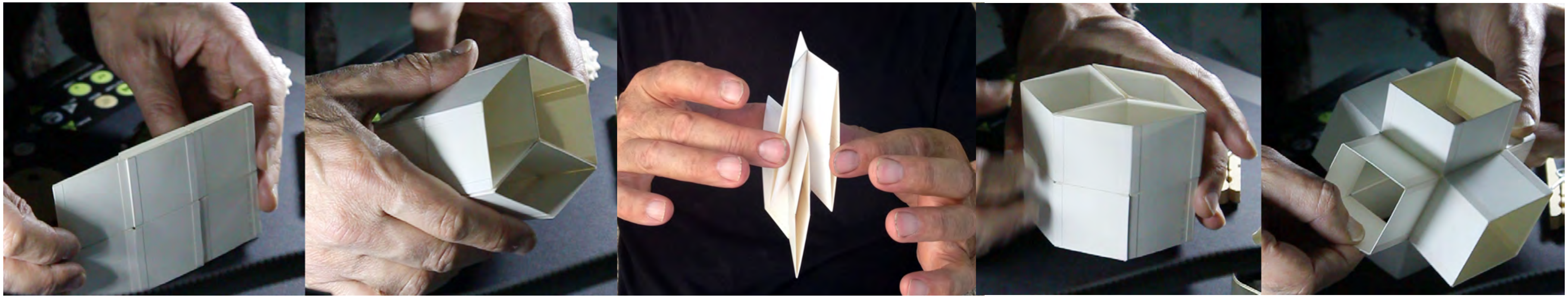
Scarpa, G., *Models of rotational geometry*, 1978, p. 93.

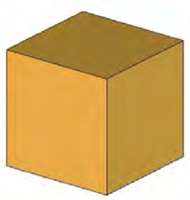
TWO
TRANSFORMABLE SHAPES:
“FIGURE TRASFORMABILI”
1996

*"I hope you like the transformable shape you will find in the package.
But it was something else that I wanted to send you, a paper object that I built in three different versions but which still has more than one problem to be resolved."*

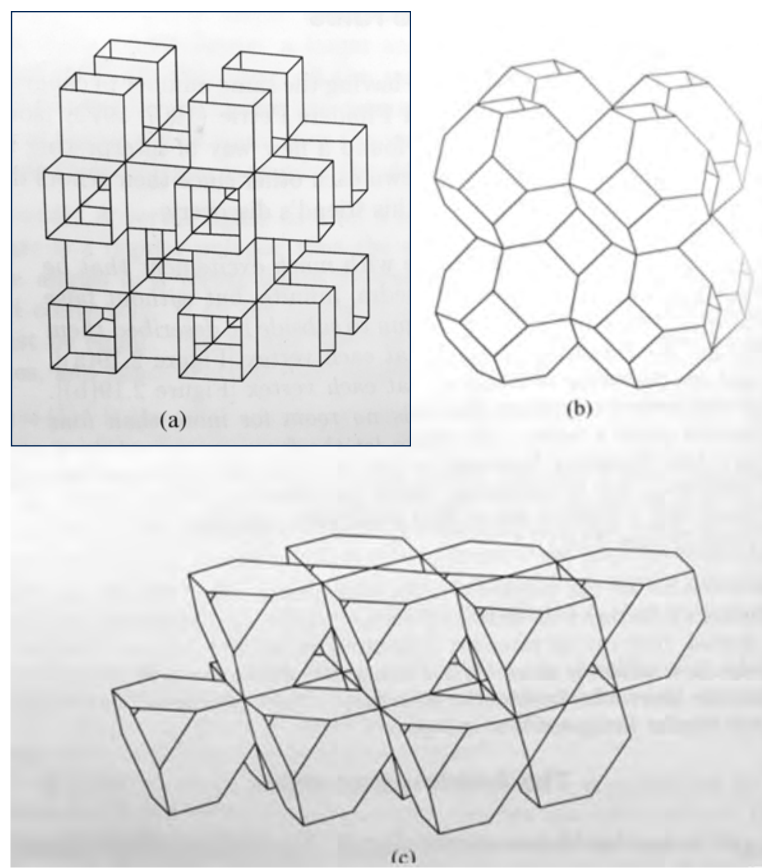
July 28, 1996

SCARPA, 1996

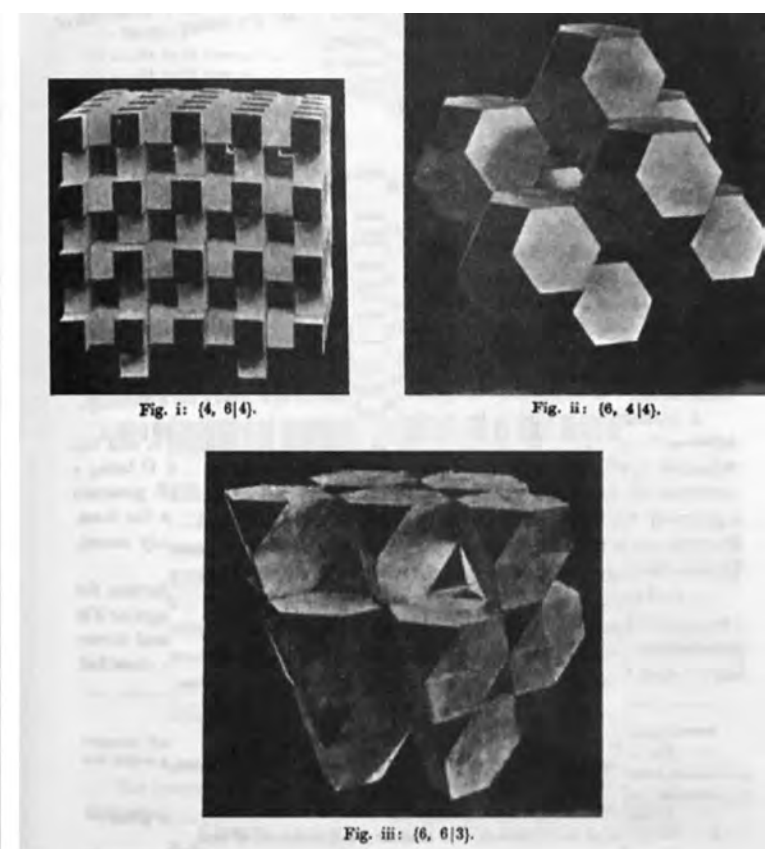




J. F. PETRIE REGULAR HONEYCOMB 1926

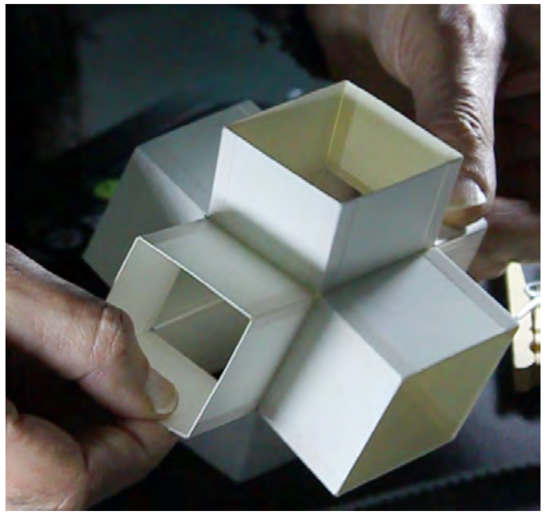


P. Cromwell, Polyhedra,
Cambridge University Press, 1997. p.79

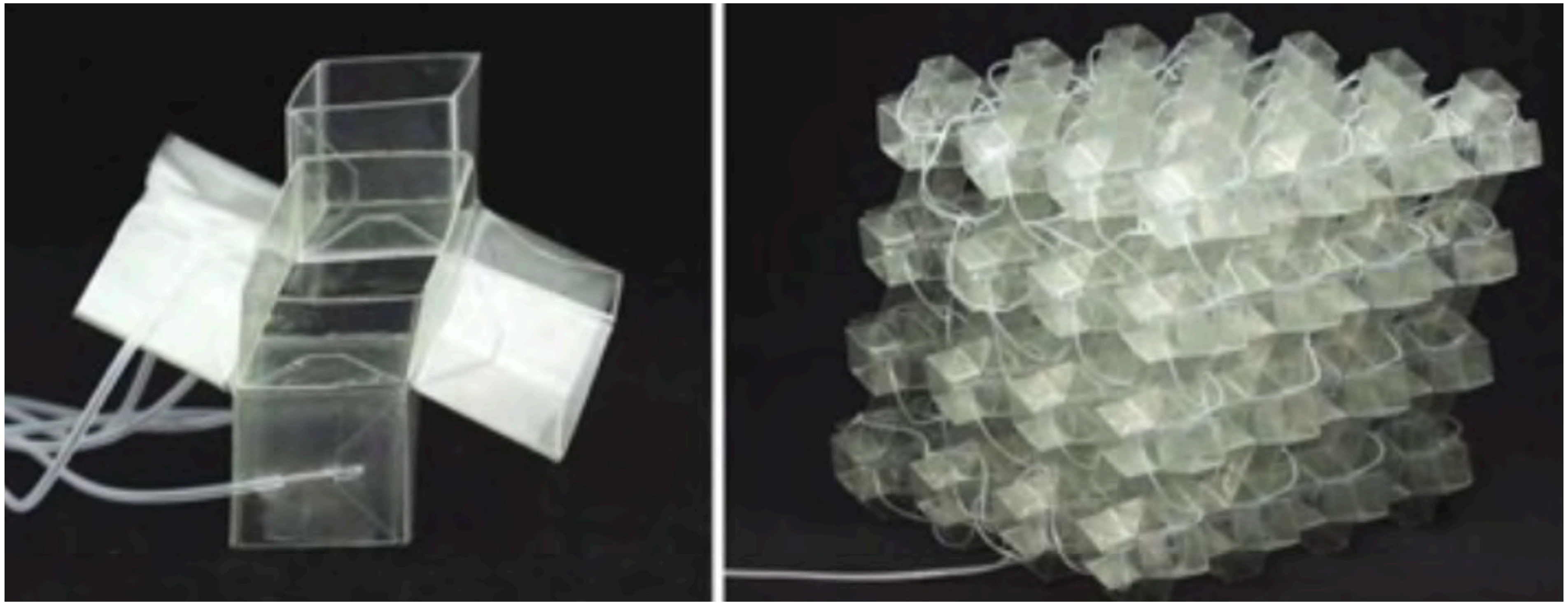


H.S.M. Coxeter, Regular Skew Polyhedra in Three and Four Dimensions, and Their Topological Analogues, *Proc. London Math. Soc.* (series 2) 43 (1937) pp33-34

SCARPA, 1996

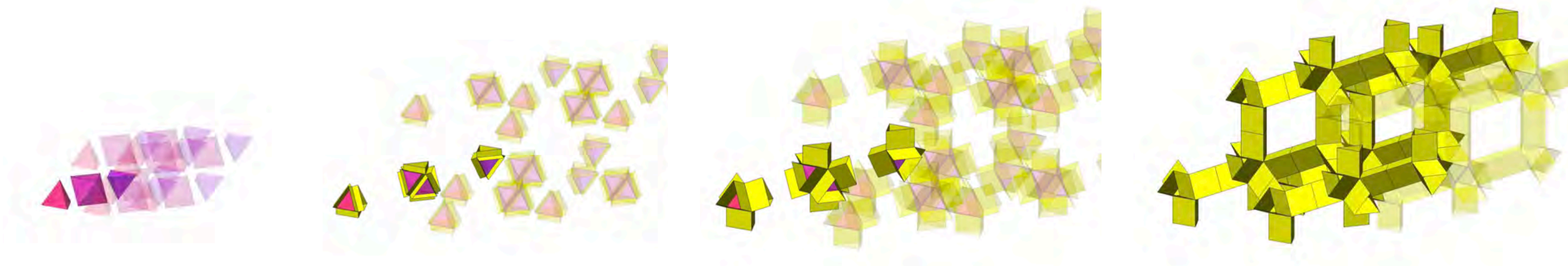


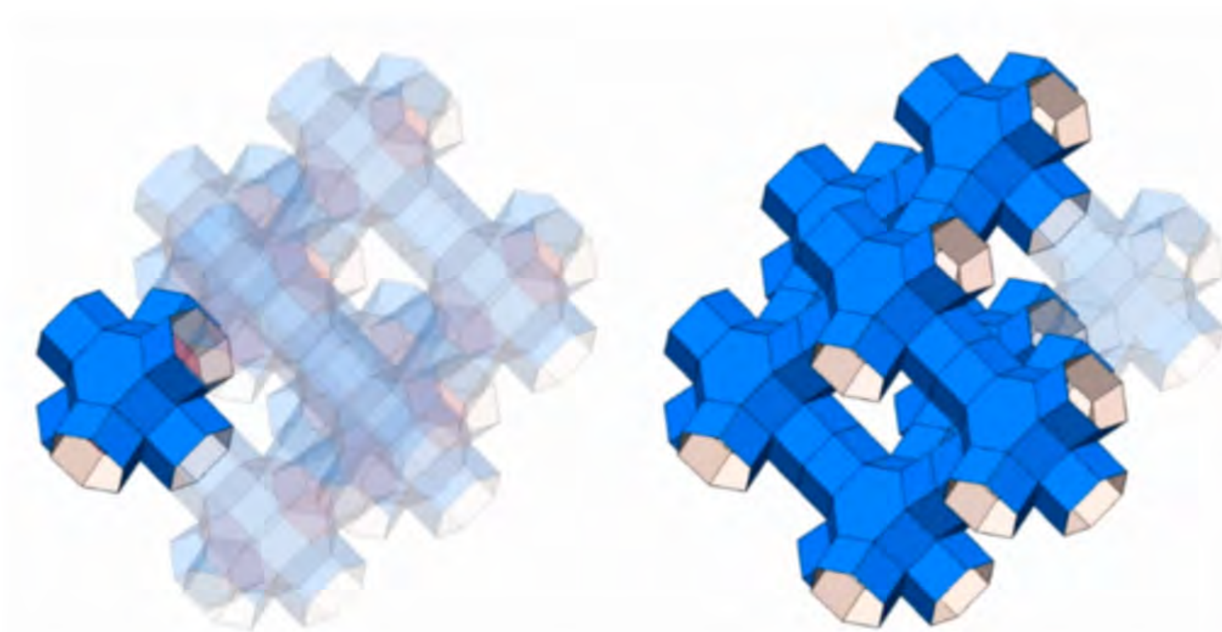
OVERVELDE ET AL, 2016

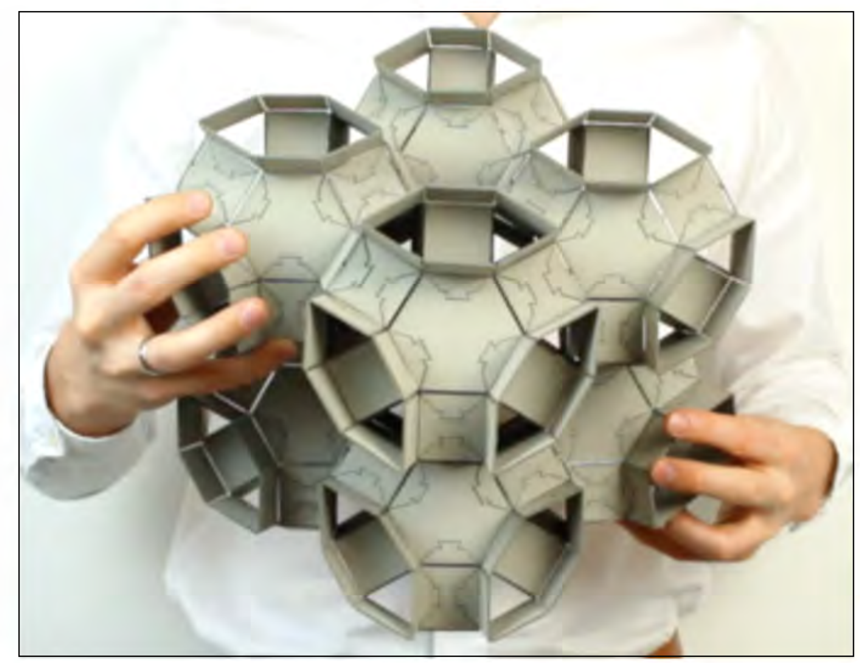


Overvelde, J. T. B. et al, A three-dimensional actuated origami-inspired transformable metamaterial with multiple degrees of freedom. *Nat. Commun*. 7, 10929 (2016)

OVERVELDE ET AL, NATURE, 2017

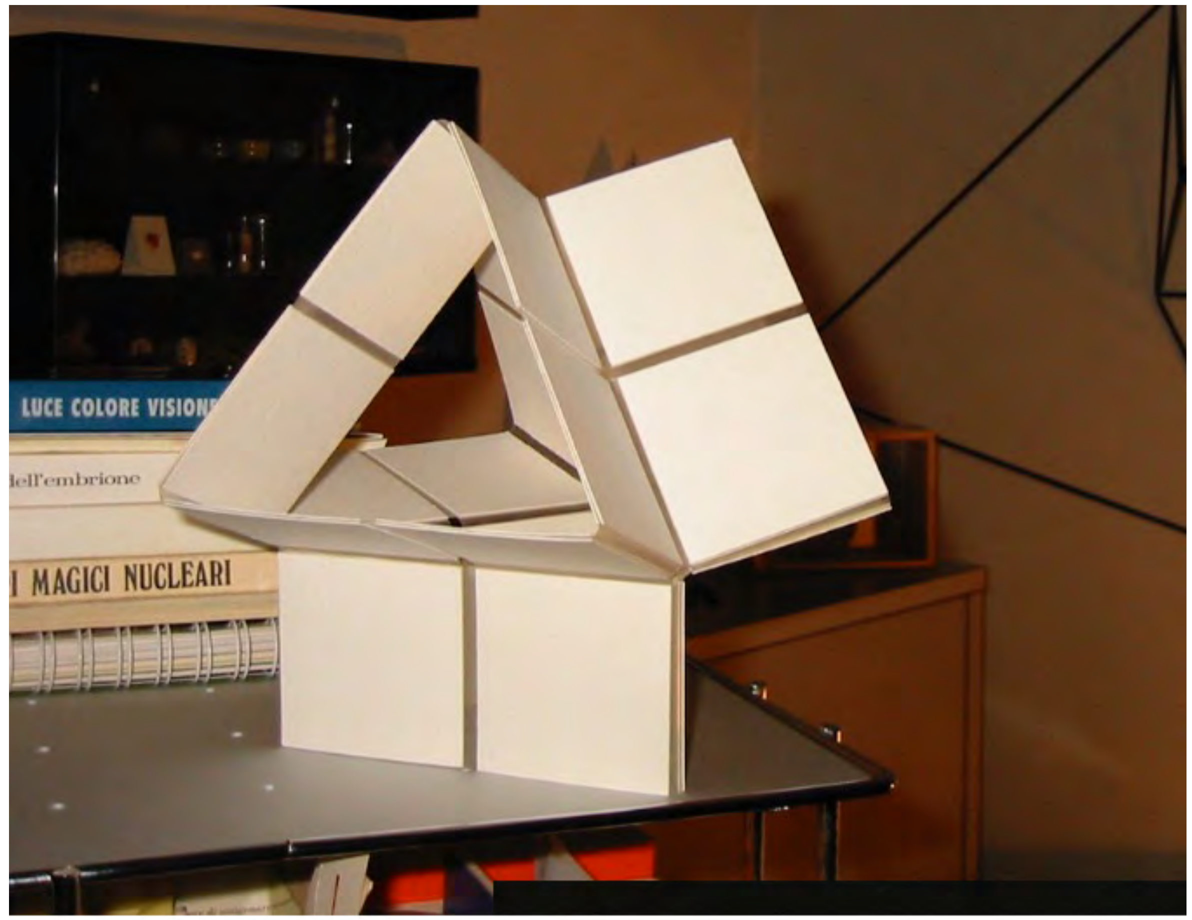


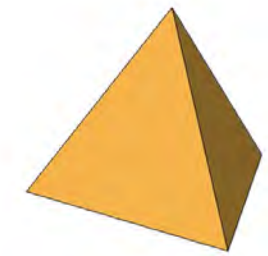




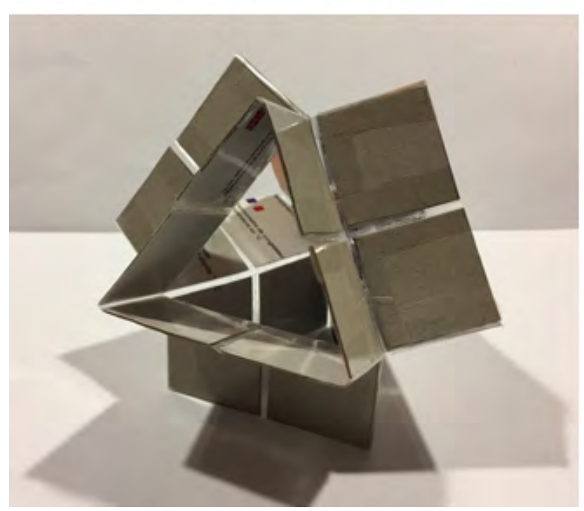
Overvelde, J., Weaver, J., Hoberman, C. et al.
Rational design of reconfigurable prismatic architected materials.
Nature 541, 347–352 (2017).

G. SCARPA – SPLIT EXTRUSION TETRAHEDRON – 1996





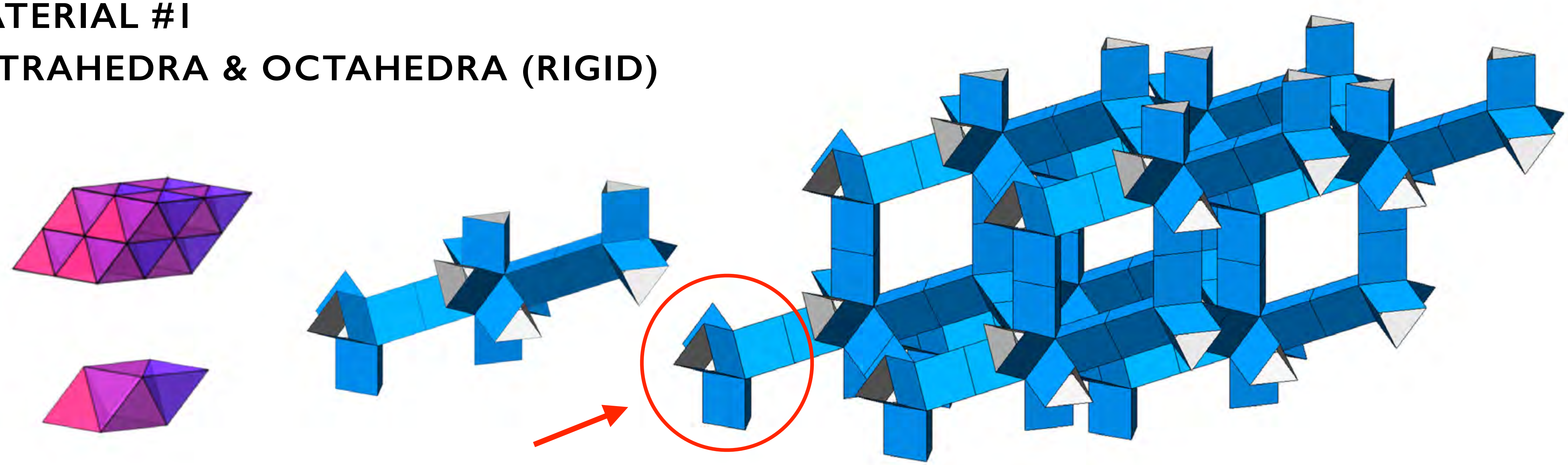
First replica by P. Trogu, 2017

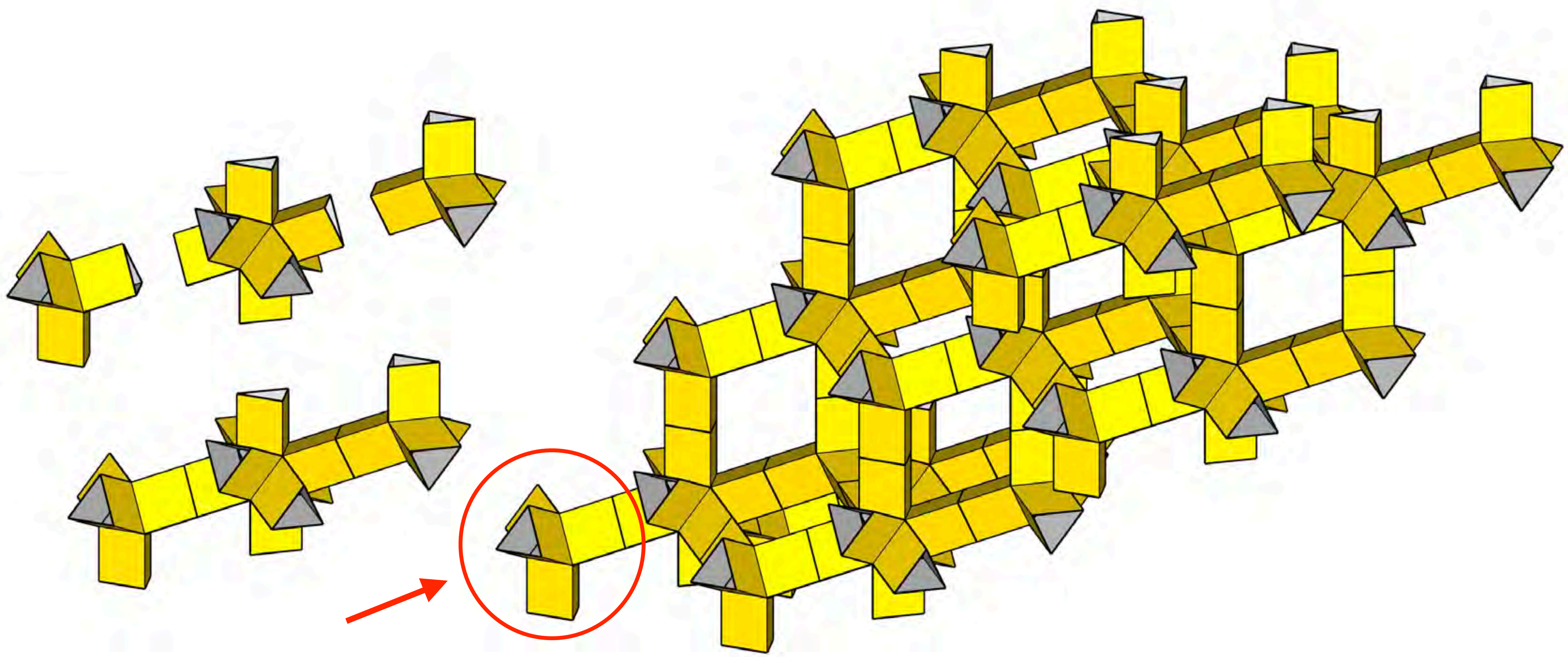


FLAT-FOLDABLE AUXETIC METAMATERIALS (PROCESS)

OVERVELDE ET AL, 2017

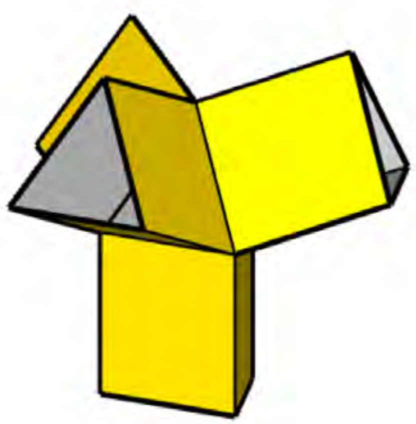
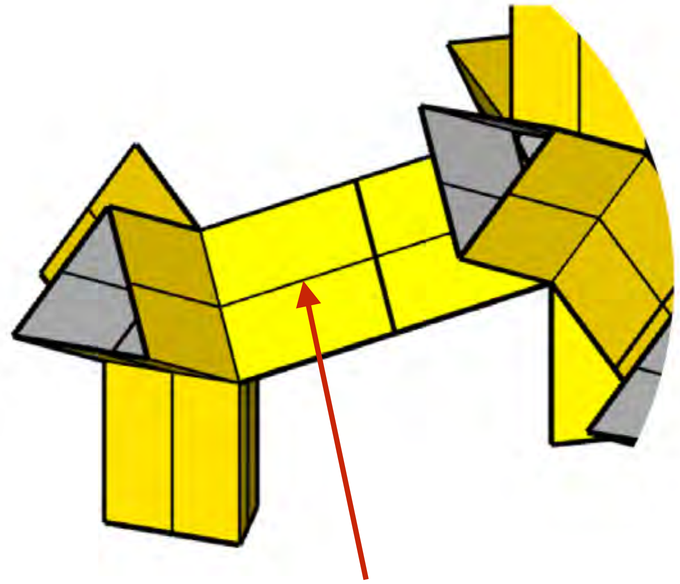
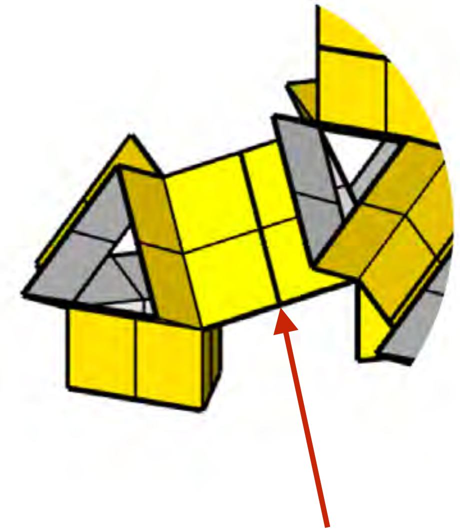
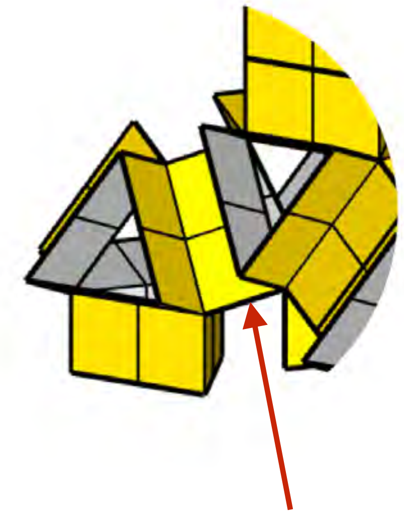
MATERIAL #1
TETRAHEDRA & OCTAHEDRA (RIGID)





TROGU, 2023

MATERIAL #1
TETRAHEDRA & OCTAHEDRA (FLAT-FOLDABLE)

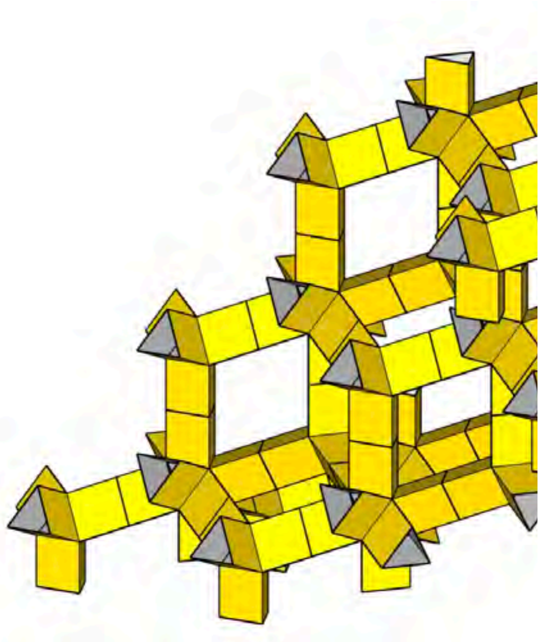
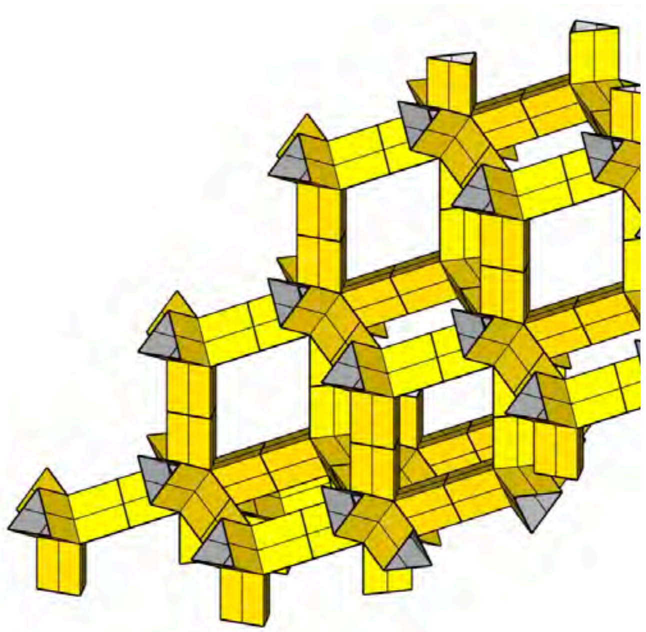
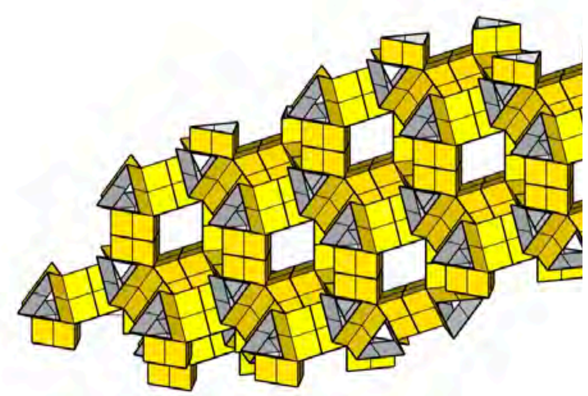
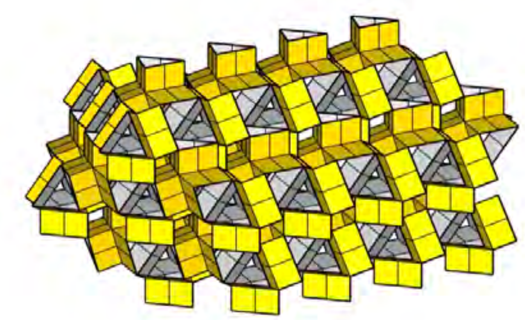





START

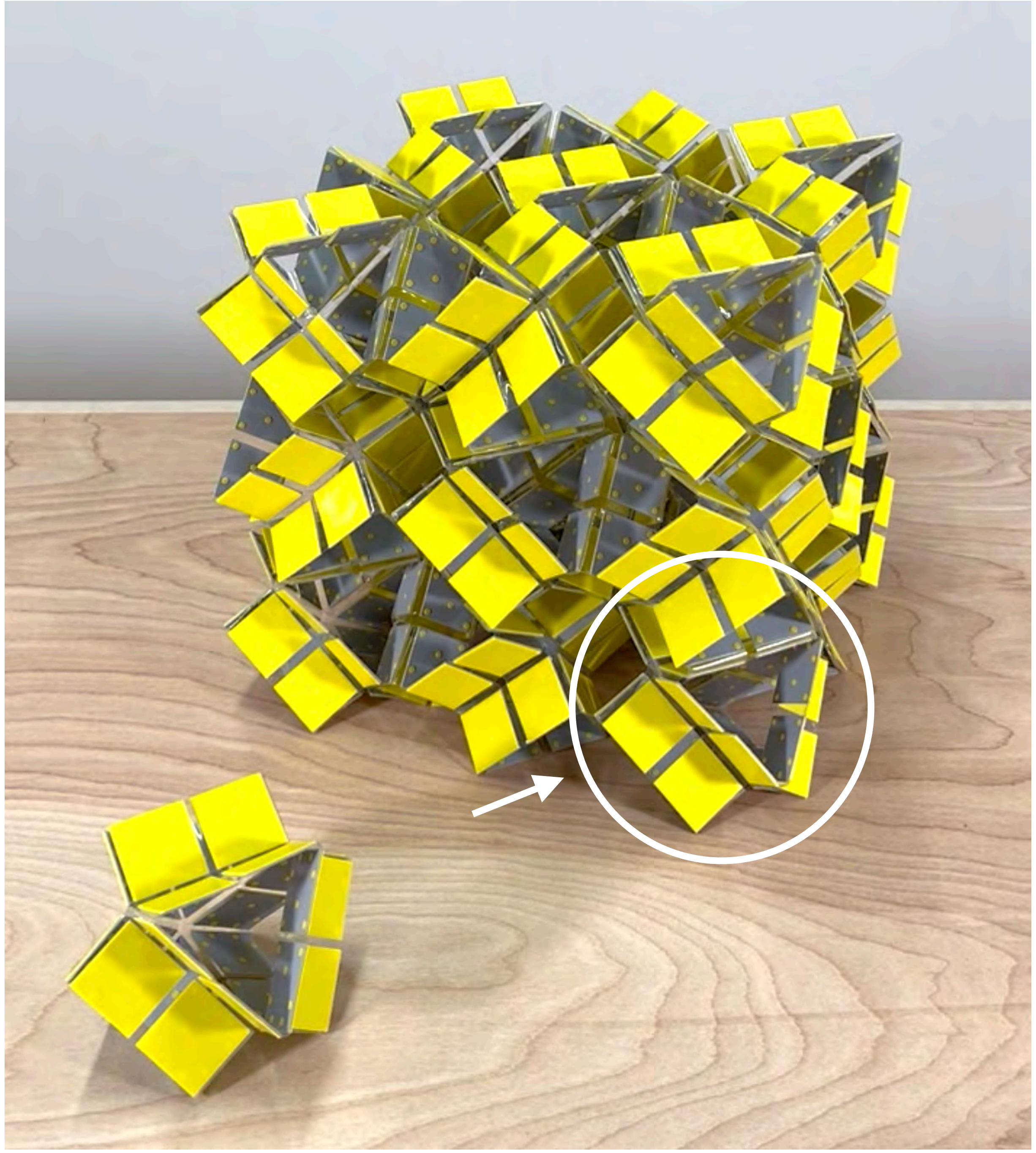
STEP 1: BISECT EXTRUDED PRISM WALLS

STEP 2: HALVE THE EXTRUSION LENGTH

STEP 3: "PUSH-FIT" THE EXTRUDED PRISMS

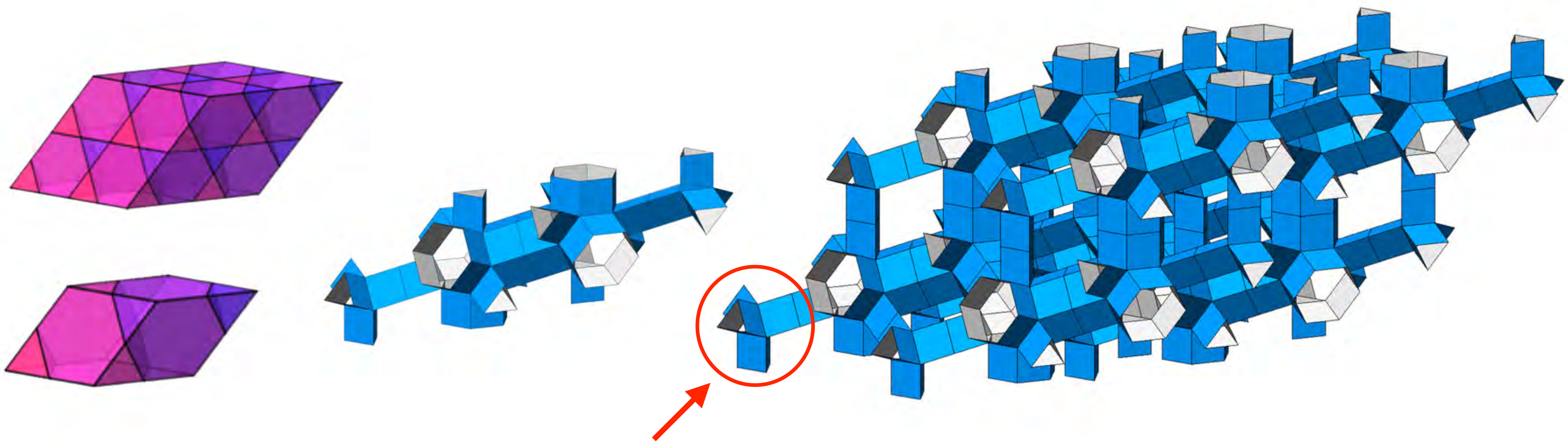
MATERIAL #1
TETRAHEDRA & OCTAHEDRA (FLAT-FOLDABLE)



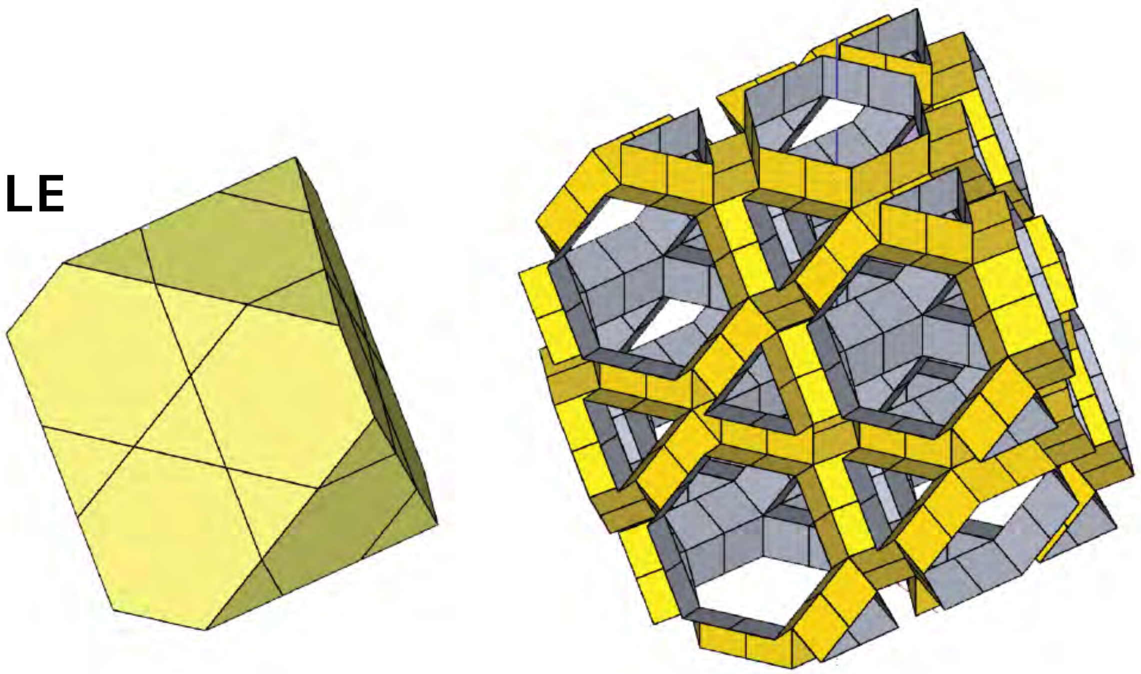
(Model fabrication:
E. Montano, J. Minnick,
D. Munechica, J. Cuellar,
P. Trogu, 2023)

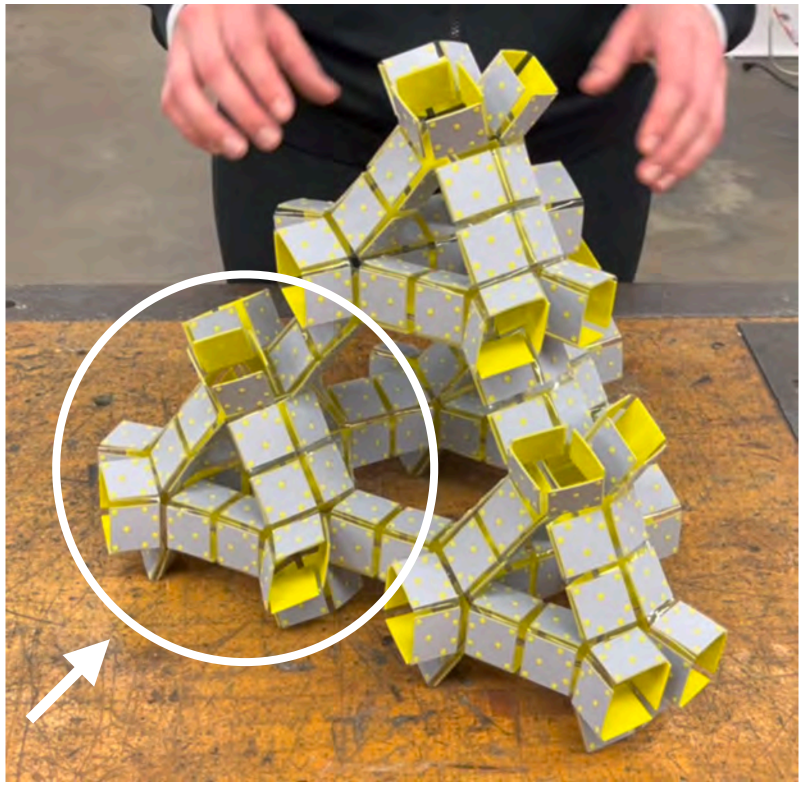
OVERVELDE ET AL, 2017

MATERIAL #6
TETRAHEDRA & TRUNCATED TETRAHEDRA (RIGID)



TROGU, 2023
MATERIAL #6
FLAT-FOLDABLE

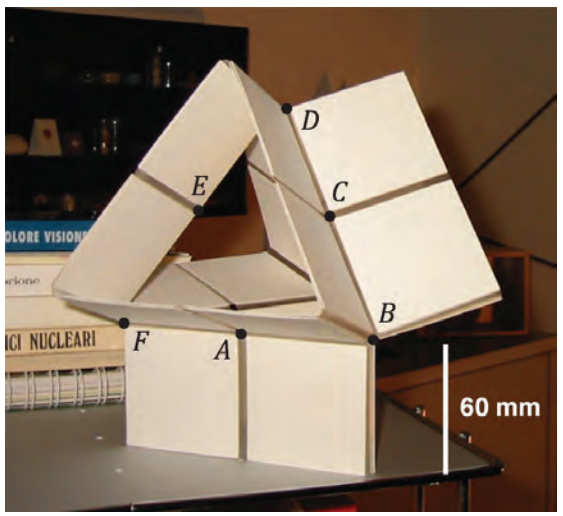


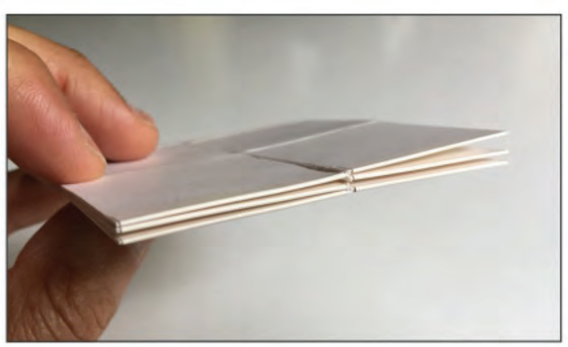


Model fabrication: E. Montano, J. Minnick, D. Munechica, J. Cuellar, P.Trogu, 2023

EXTRUDED TETRAHEDRON: FROM NON RIGIDLY FOLDABLE TO RIGIDLY FOLDABLE

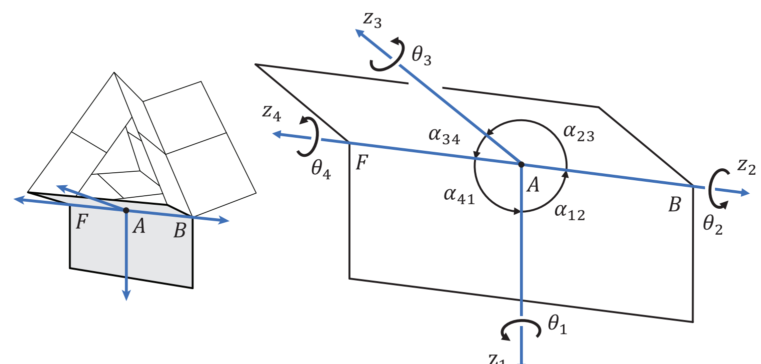
Scarpa's transformable model (1996)



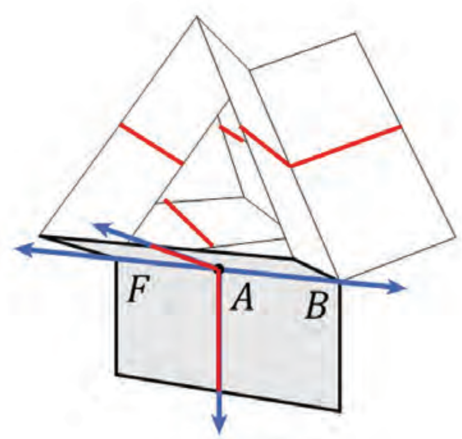


It is flat-foldable

Rigid foldability analysis



D-H notation of the four-crease vertex *A*.



inactive creases

It is not rigidly foldable

Conditions:

$$\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = \frac{\pi}{2}$$

Closure equation (D-H):

$$\boldsymbol{Q}_{21}\boldsymbol{Q}_{32} = \boldsymbol{Q}_{41}\boldsymbol{Q}_{34}$$

where

$$\boldsymbol{Q}_{(i+1)i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_{i(i+1)}\sin\theta_i & \sin\alpha_{i(i+1)}\sin\theta_i \\ \sin\theta_i & \cos\alpha_{i(i+1)}\cos\theta_i & -\sin\alpha_{i(i+1)}\cos\theta_i \\ 0 & \sin\alpha_{i(i+1)} & \cos\alpha_{i(i+1)} \end{bmatrix}$$

$$\boldsymbol{Q}_{i(i+1)} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\cos\alpha_{i(i+1)}\sin\theta_i & \cos\alpha_{i(i+1)}\cos\theta_i & \sin\alpha_{i(i+1)} \\ \sin\alpha_{i(i+1)}\sin\theta_i & -\sin\alpha_{i(i+1)}\cos\theta_i & \cos\alpha_{i(i+1)} \end{bmatrix}$$

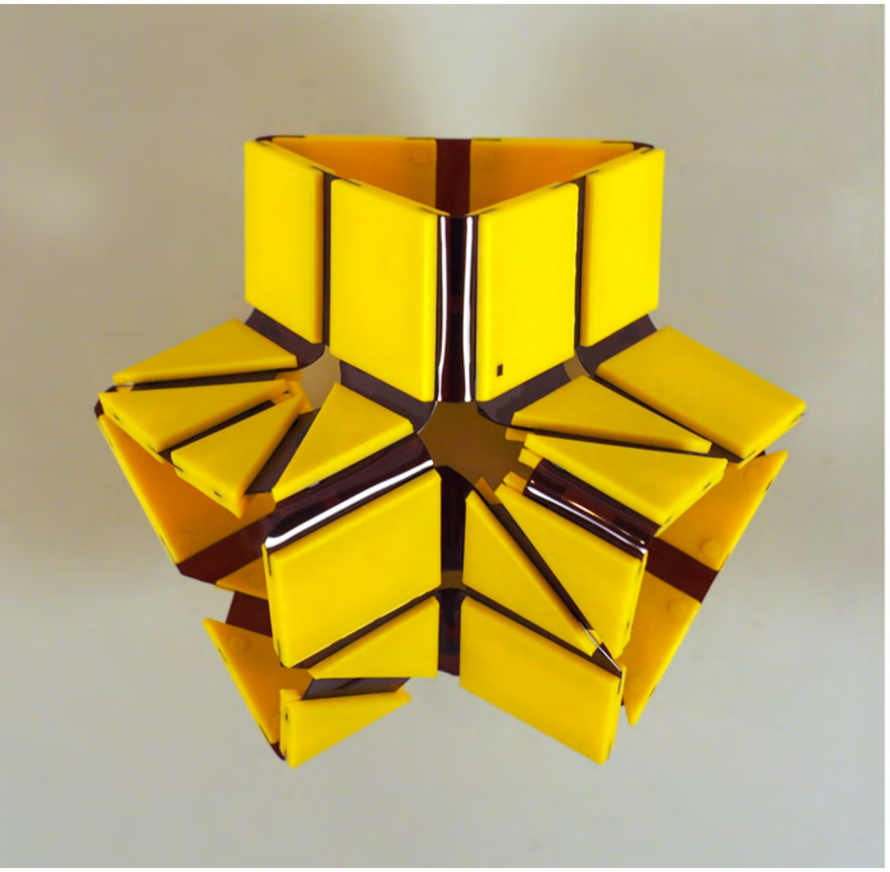
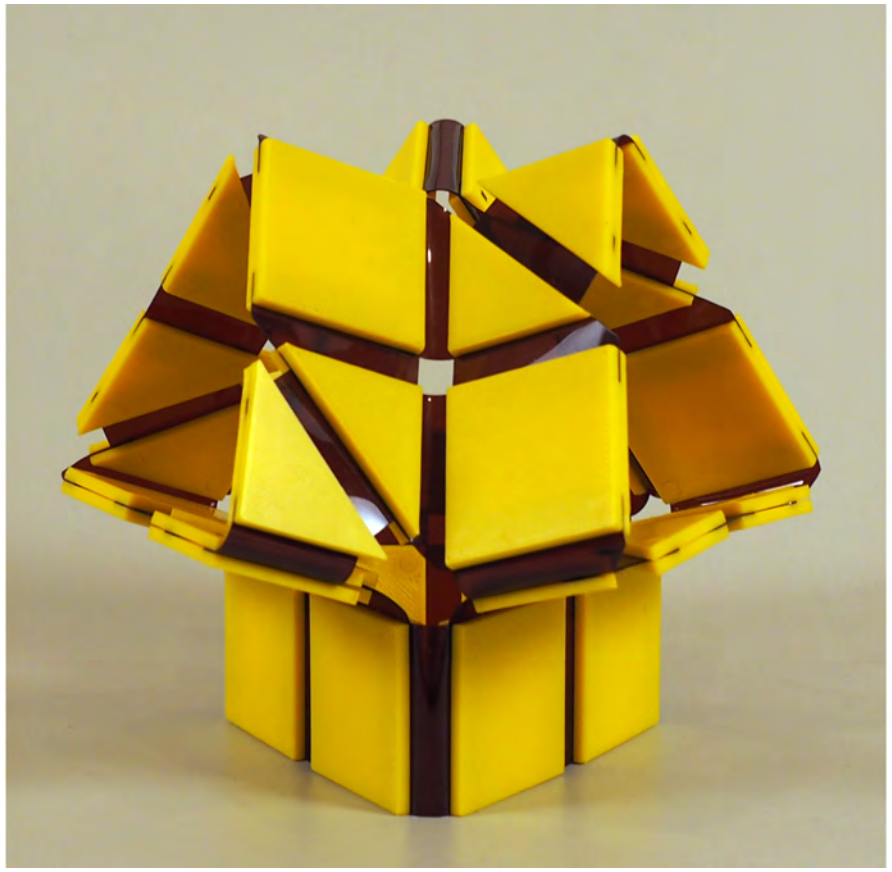
Solution:

$\theta_1 = \theta_3 = 0,\ \theta_2 = \theta_4$;
or $\theta_2 = \theta_4 = 0,\ \theta_1 = \theta_3$.

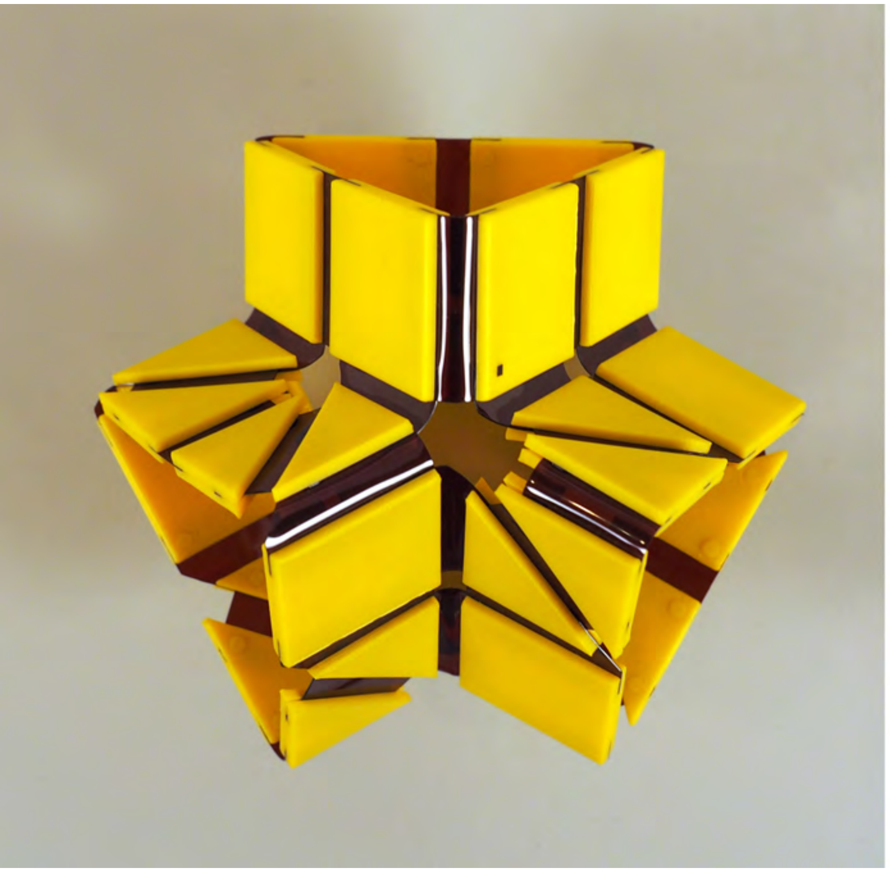
SCARPA'S ORIGINAL SHAPE: NON-RIGIDLY FOLDABLE

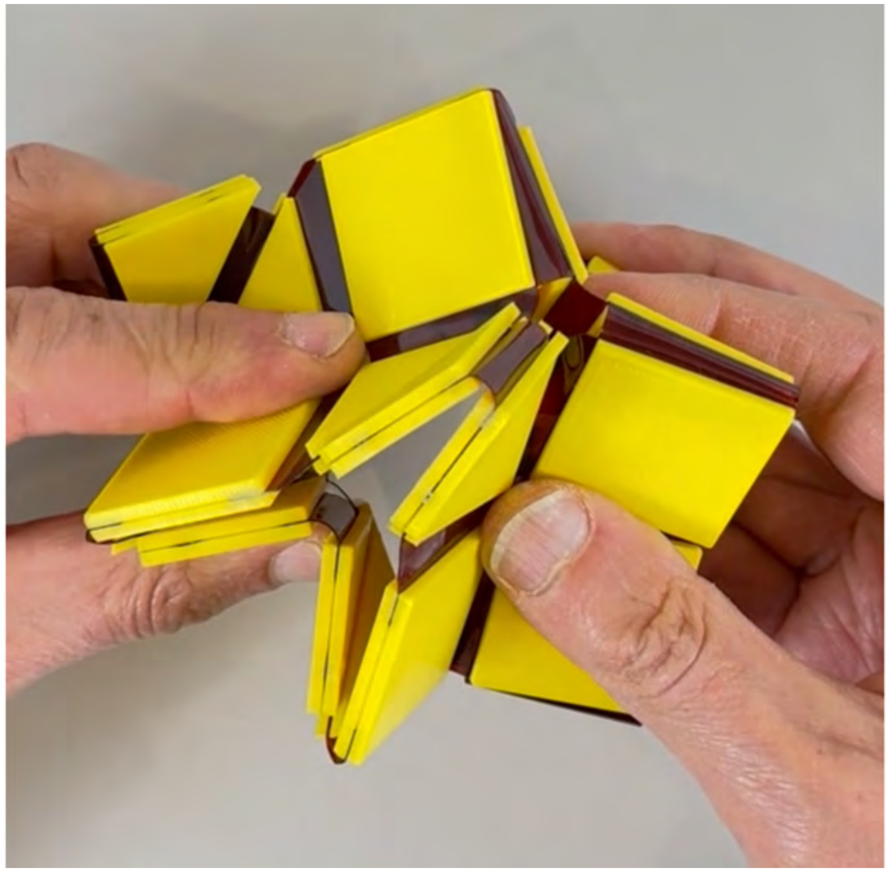
Model fabrication: H. Feng, W. Shi, P. Trogu, E. Montano, J. Minnick, G. Montalvo, T. Casanova, 2023

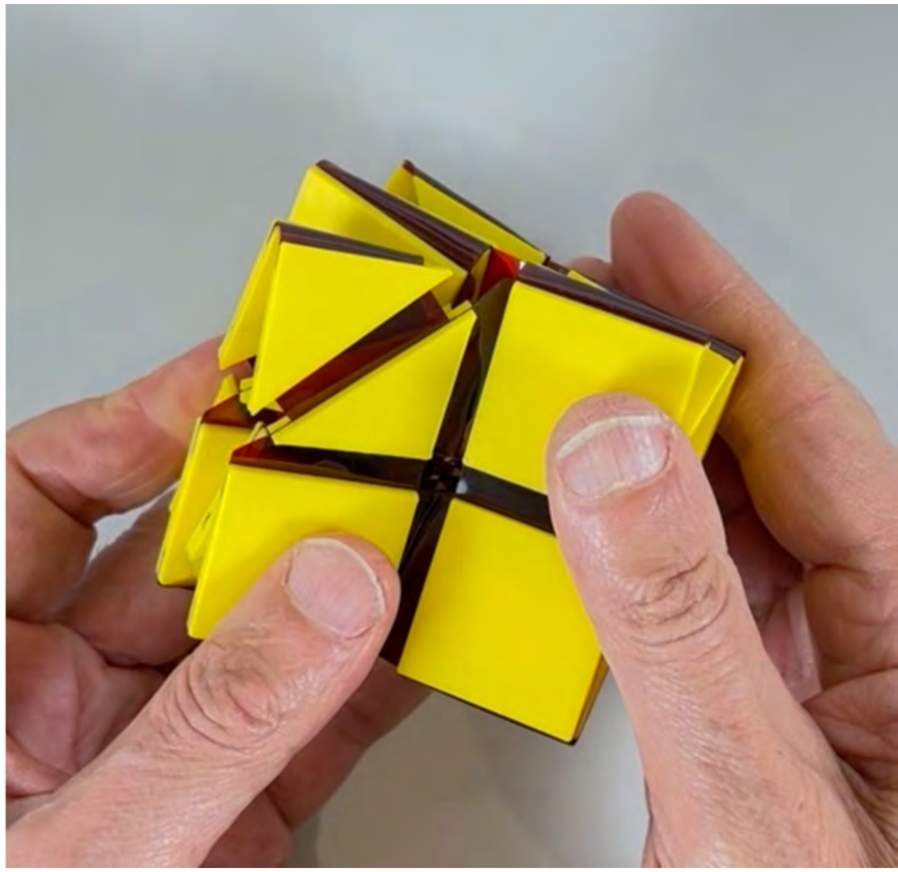
MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES


MODIFIED SHAPE: RIGIDLY FOLDABLE WITH ADDITIONAL CREASES



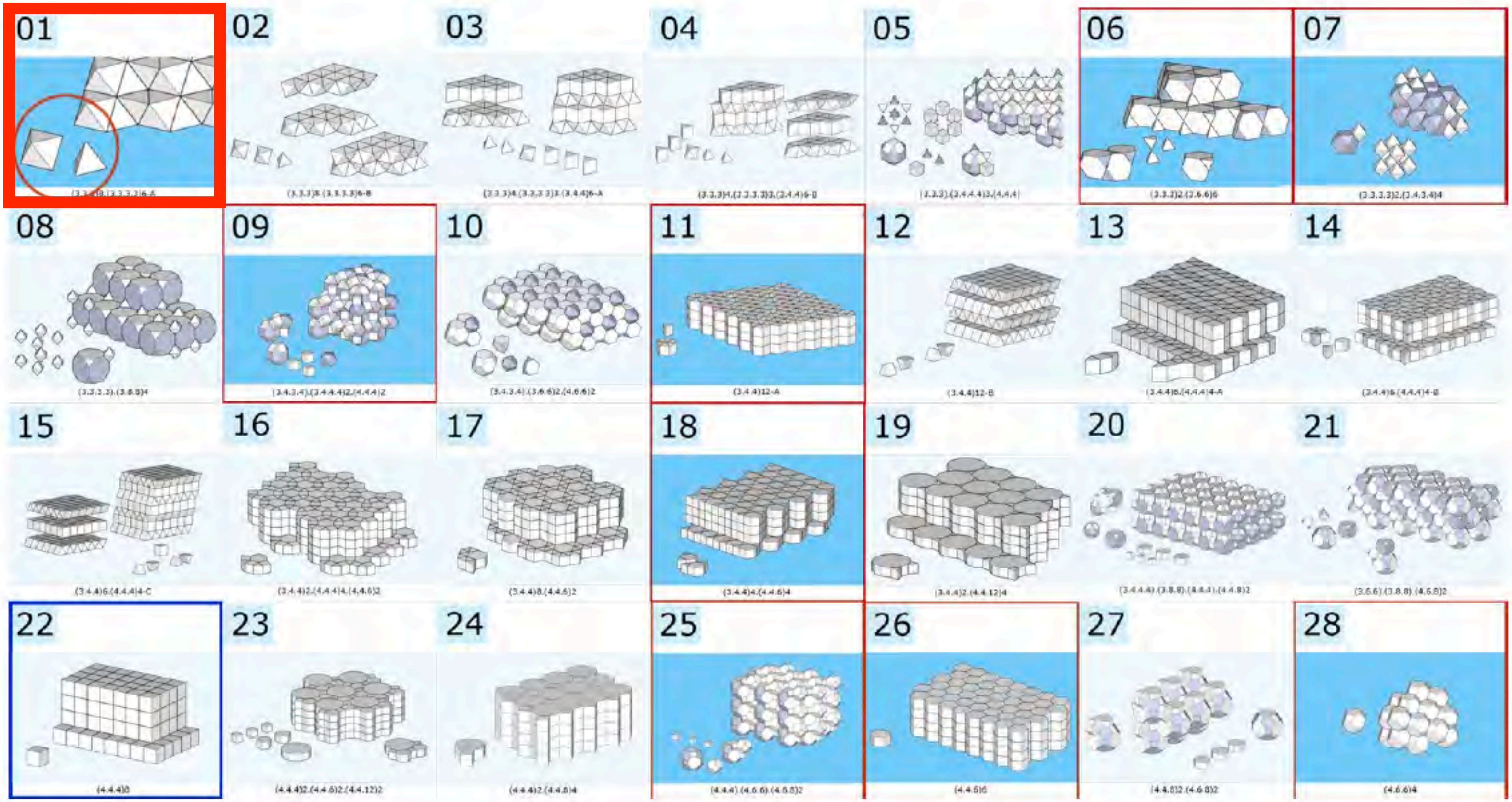




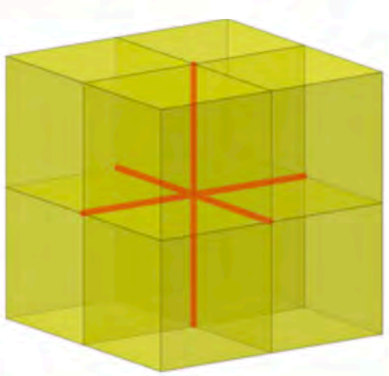
MATERIAL #1

HOW DOES THE ORIGINAL SHAPE FOLD?

UNIFORM TILINGS OF 3-SPACE*



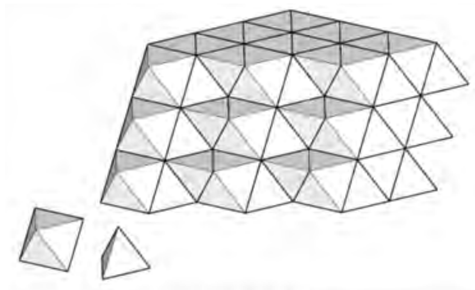
*Grünbaum, 1994
"Vertex neighborhoods" are all the same.

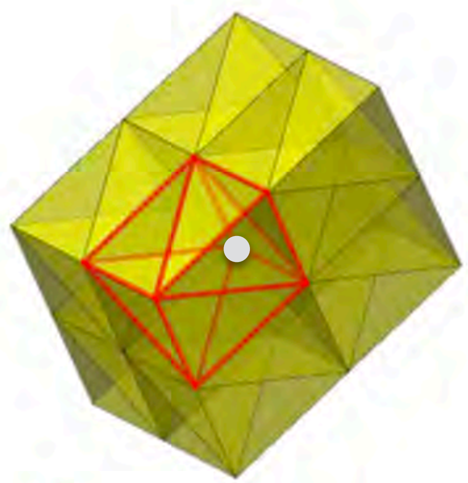


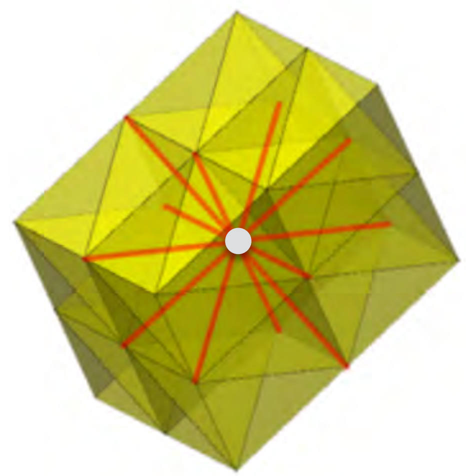
Vertex neighborhood in tiling #22 (cubes)

VERTEX NEIGHBORHOOD

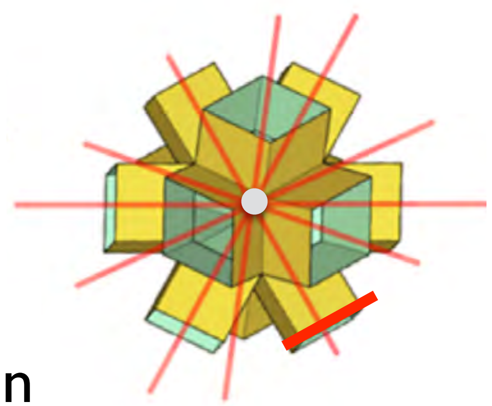
Uniform tiling #1





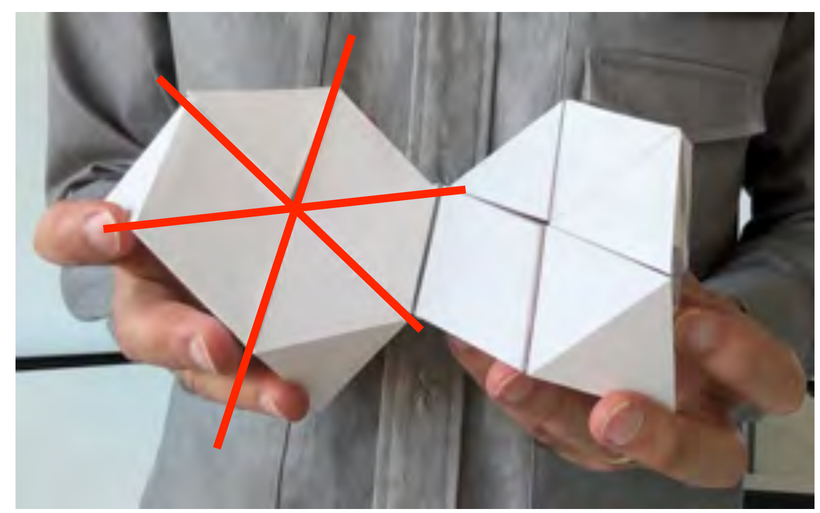


Octahedron Tetrahedron



Extruded rhombic dodecahedron (twelve prisms)

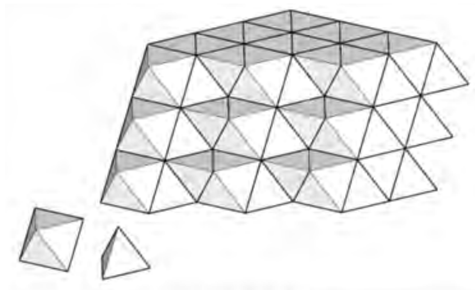
Eight tetrahedra and six tetrahedra meet at a vertex.

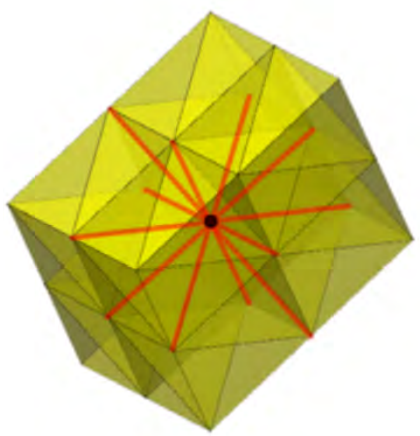


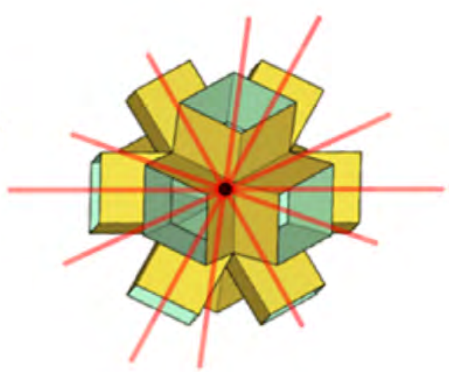
Common edges form a six-axis star.

BYPRODUCT POLYHEDRA

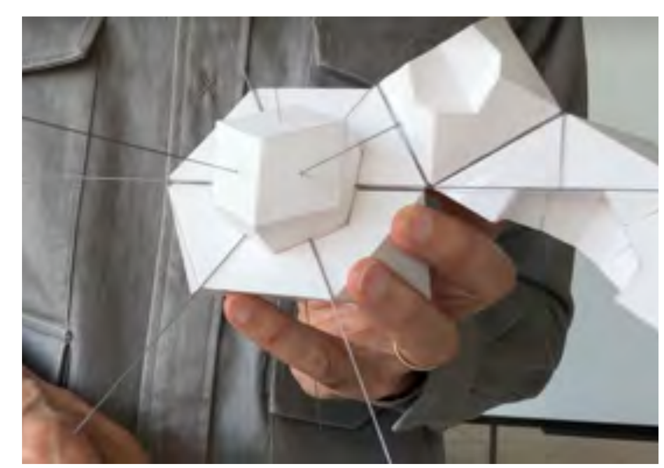
Uniform tiling #1



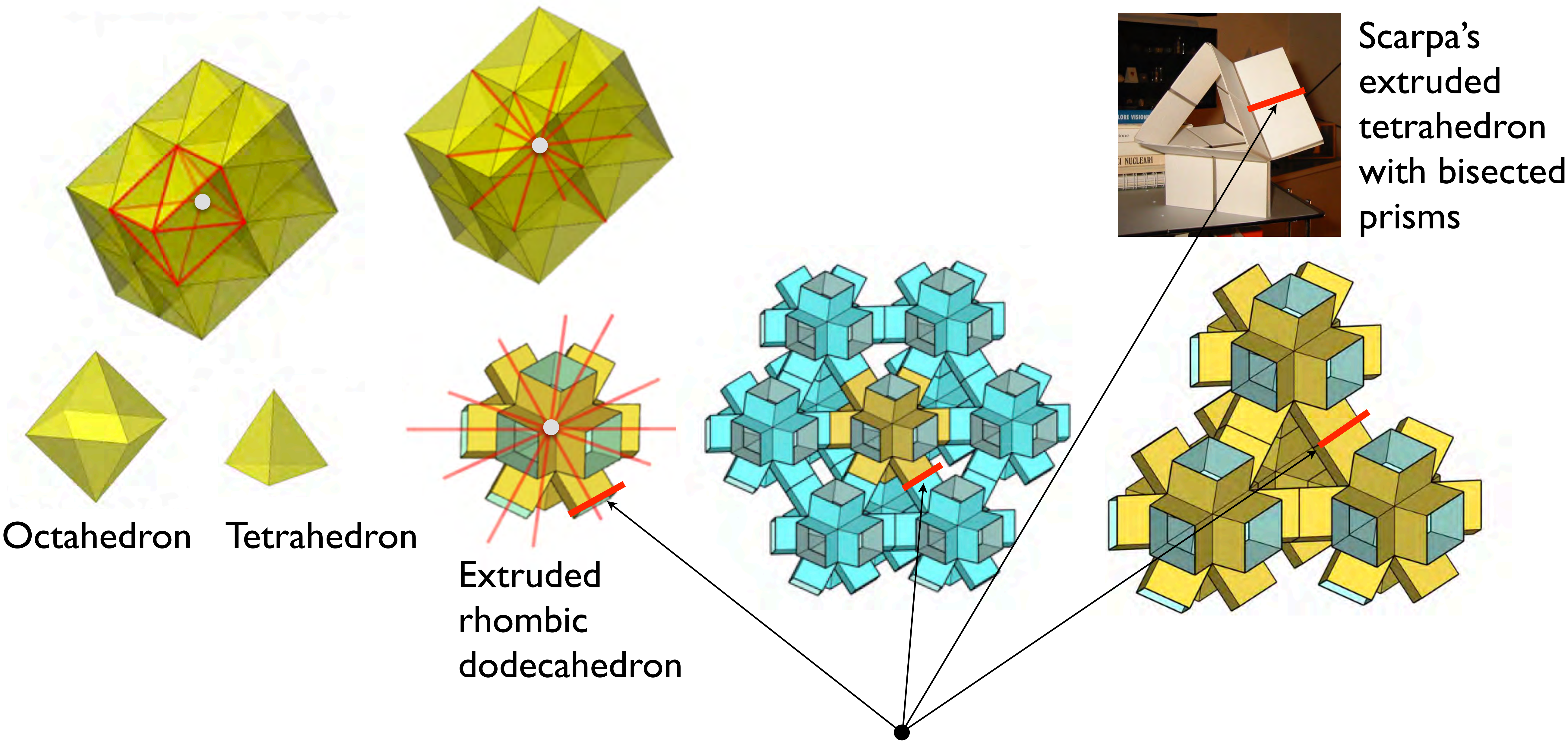




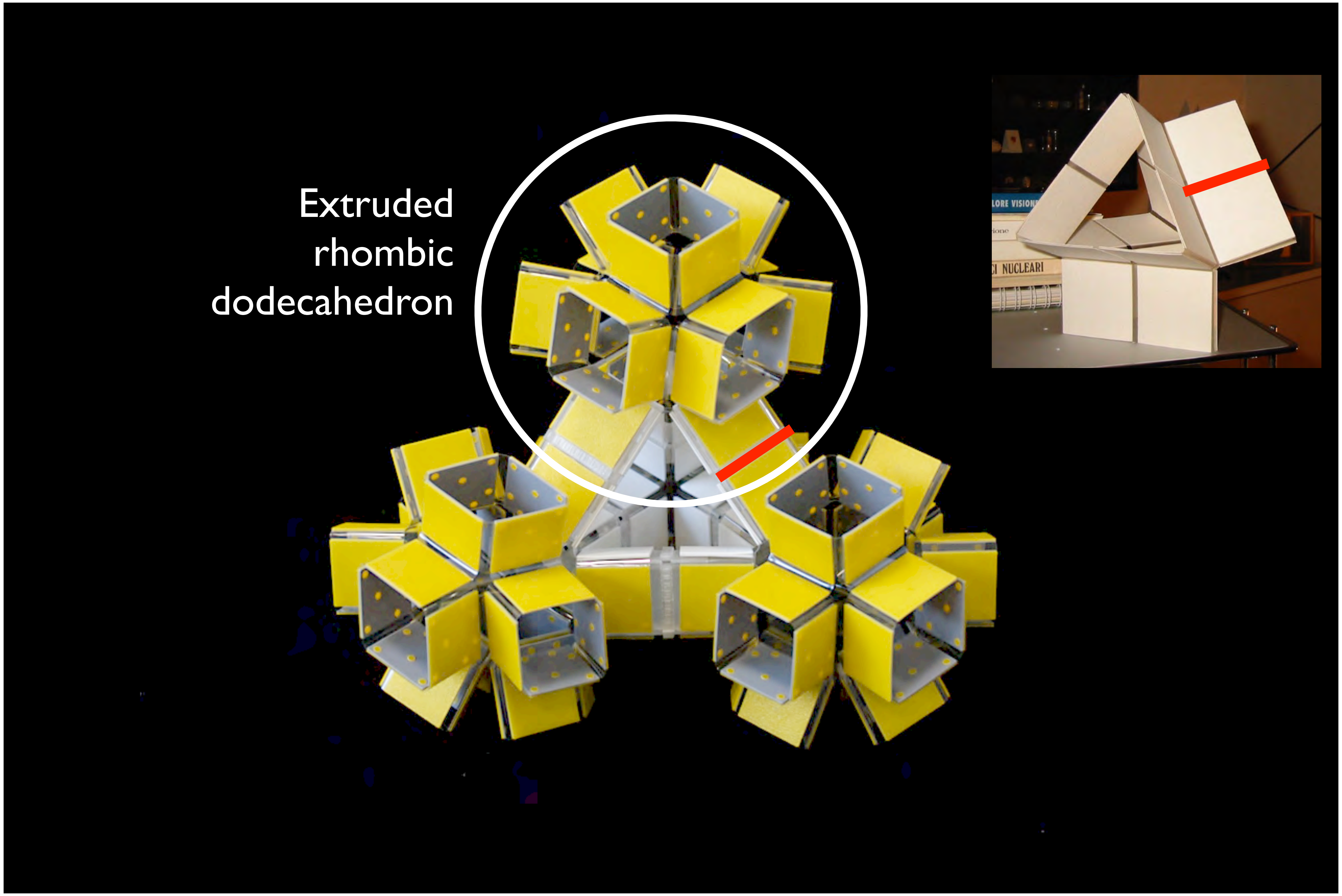
A way to predict what "byproduct polyhedra" will be generated after the base polyhedra are extruded, is to consider the dual honeycomb of the original honeycomb. Lines connecting the centers of adjacent tetrahedra and octahedra are the edges of a rhombic dodecahedron.



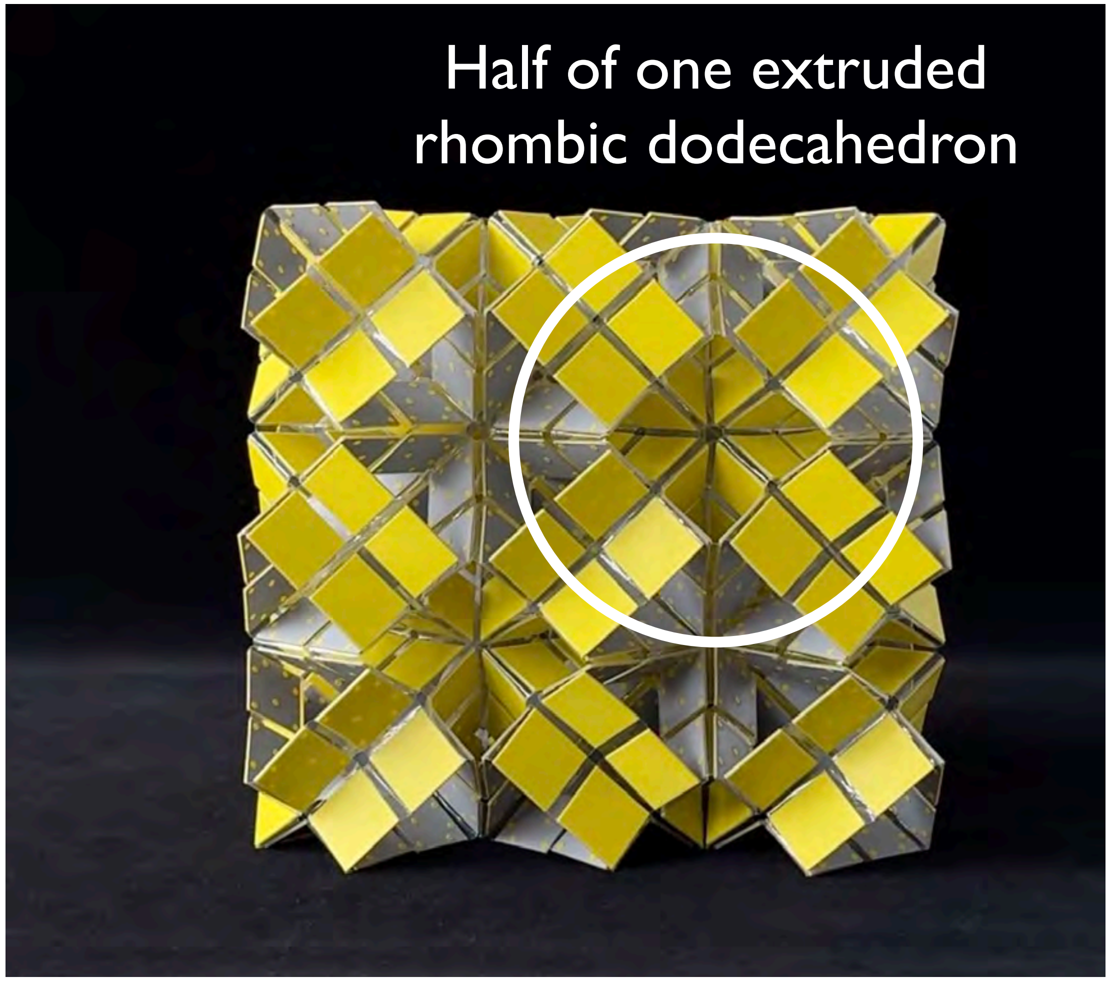
After extruding the faces of the tetrahedra and octahedra in the base tiling of material #1, rhombic dodecahedra or “byproduct polyhedra” – form at the location of the original vertex neighborhoods.

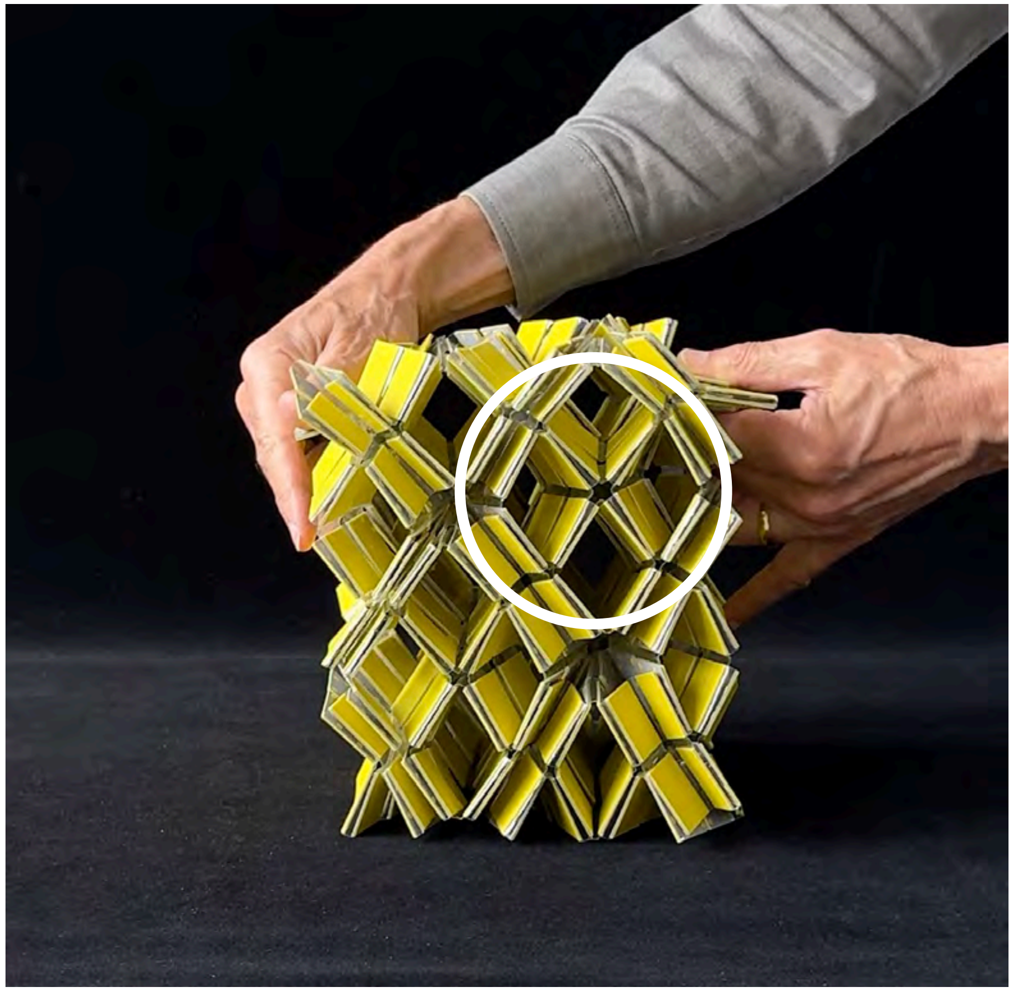


The ends of the non-split prisms in the rhombic dodecahedra correspond to the bisections in Scarpa’s original tetrahedron


Extruded
rhombic
dodecahedron

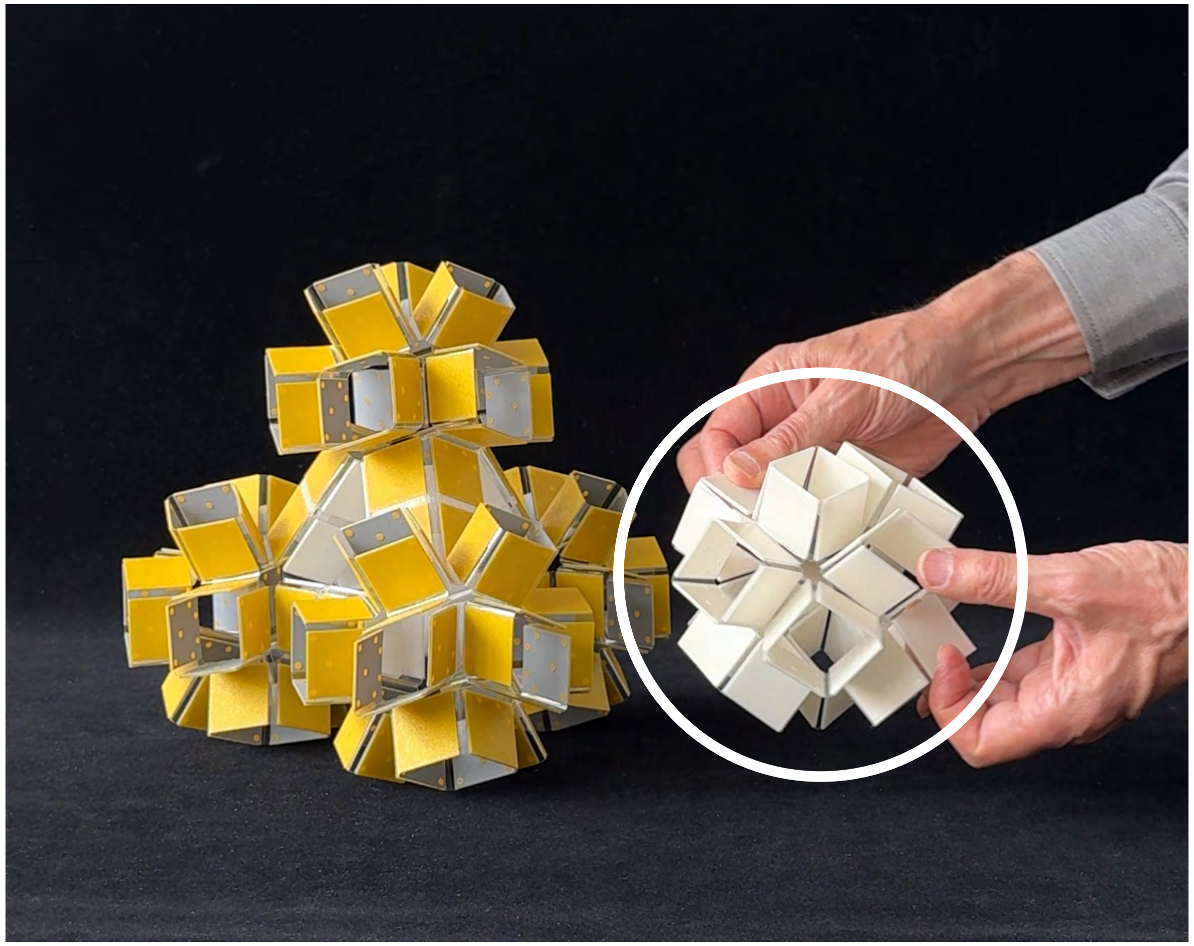
3x3x3 TETRAHEDRA MATERIAL



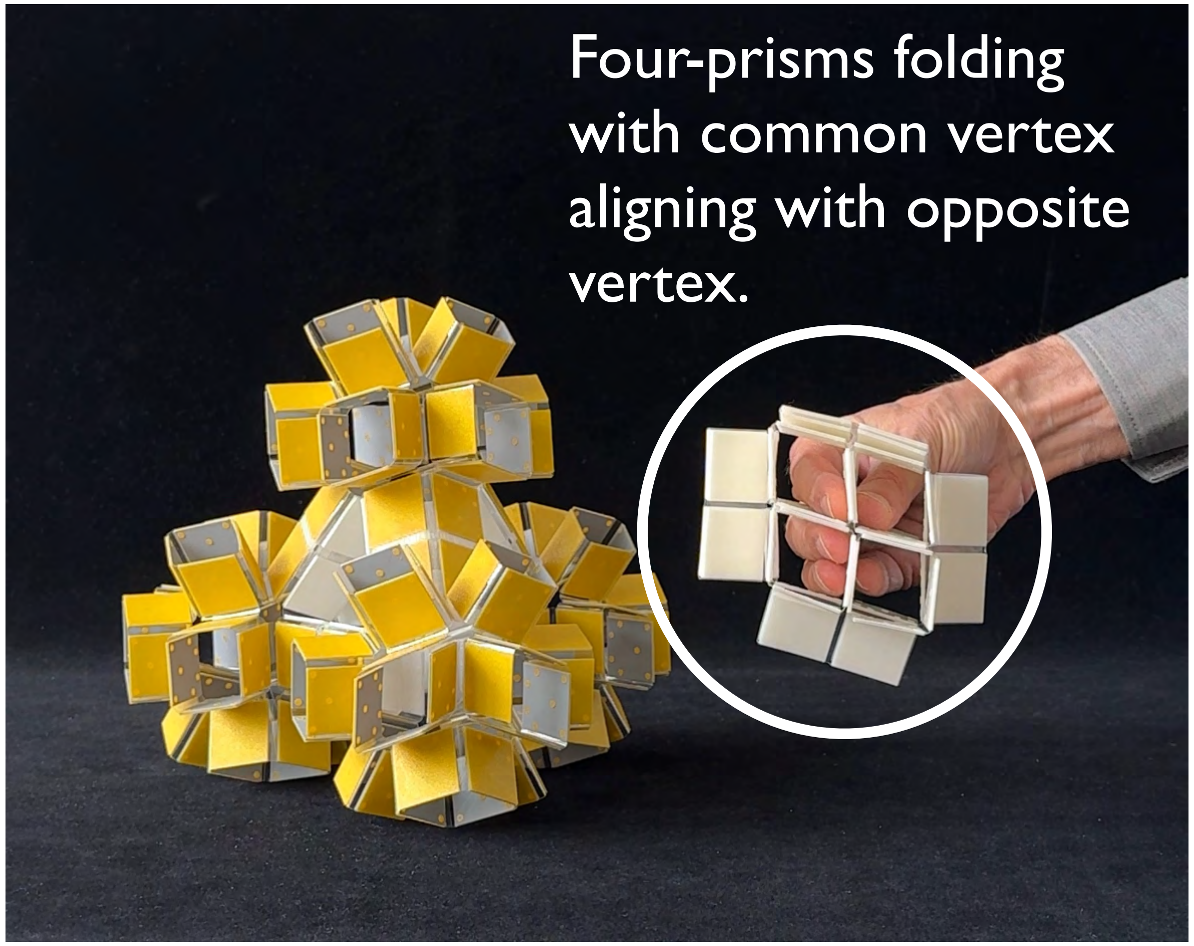


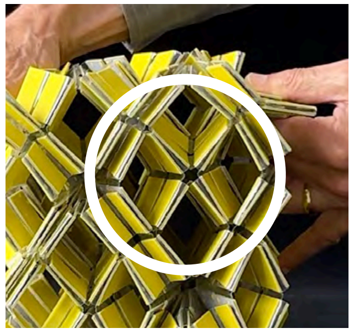
Four “channels”* are seen forming at a four-crease vertex of the rhombic dodecahedron during folding.

*(B. Kresling)



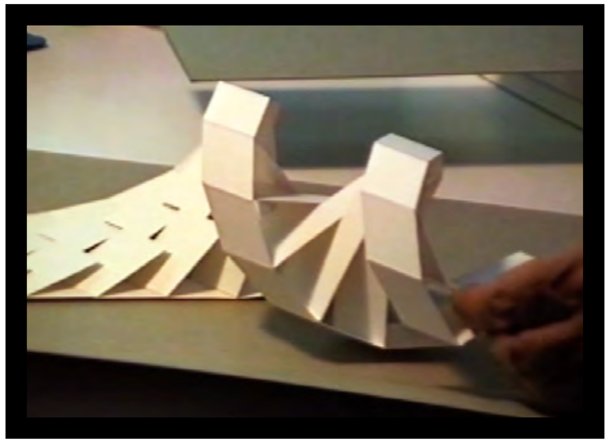
Four-crease vertex with four prisms in the rhombic dodecahedron.



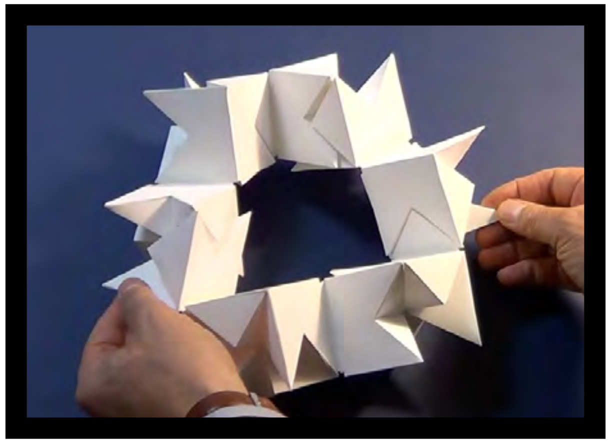


VIDEOS

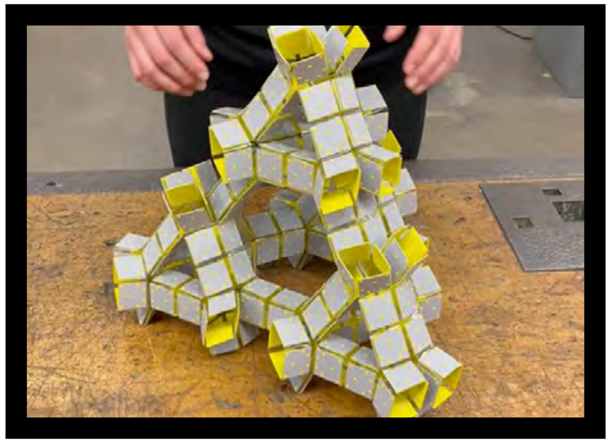
Scarpa: Aristotle's Lantern



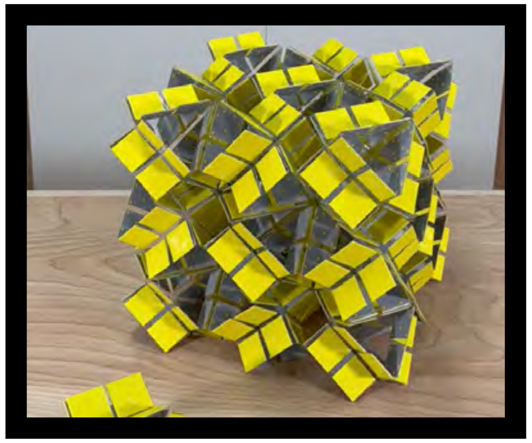
Scarpa: DNA Model



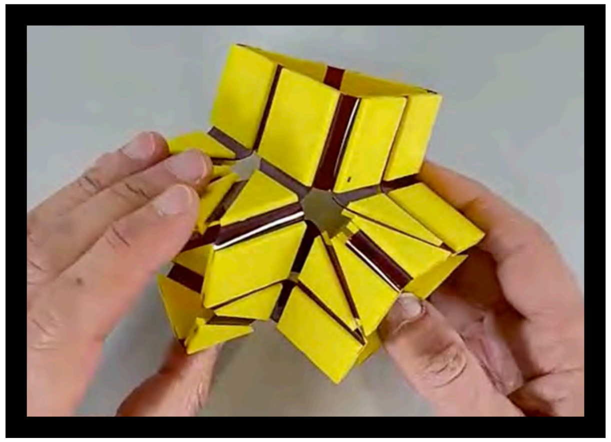
Scarpa: Hexahedral Chain



Trogu: Material #6

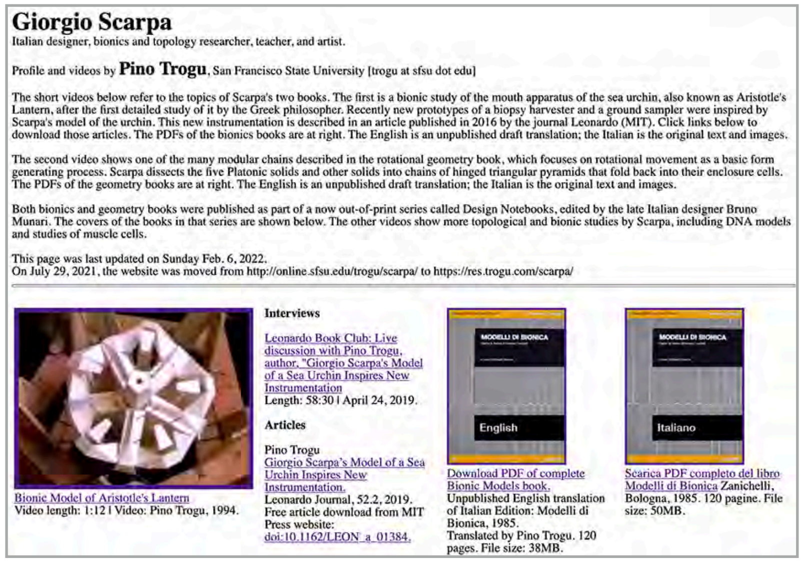


Trogu: Material #1

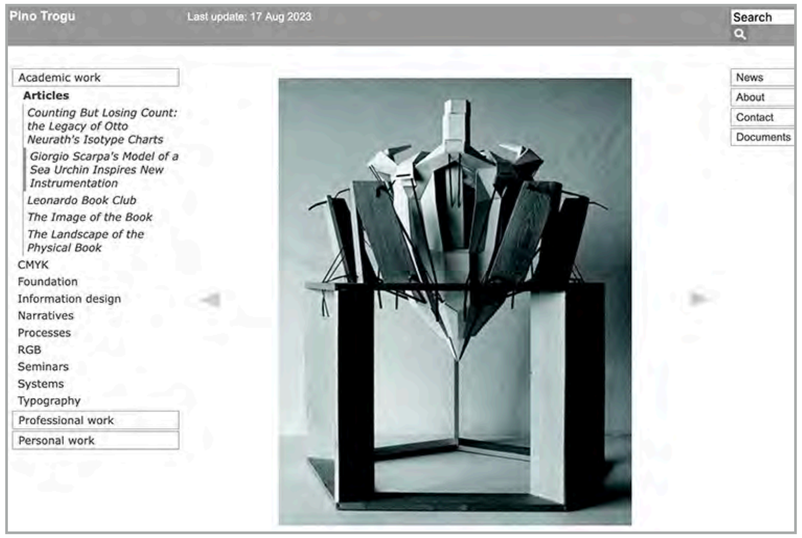


Feng, Shi, Trogu, & Dai: Auxetic Metamaterial

LINKS



res.trogu.com/scarpa



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THANK YOU!

2025 SPRING INTERNATIONAL WORKSHOP

CHINA–POLAND JOINT WORKSHOP ON ADVANCED LIGHTWEIGHT STRUCTURES AND ENERGY ABSORPTION

SOUTHEAST UNIVERSITY, NANJING — MAY 22, 2025, 11:20 AM

ROOM 304, CIVIL ENGINEERING BUILDING, JIULONGHU CAMPUS

CONTACT: TROGU@SFSU.EDU

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